Available online at www.sciencedirect.com

SCIENCE DIRECT®

Composite Structures 75 (2006) 114–120

COMPOSITE STRUCTURES

www.elsevier.com/locate/compstruct

Bending of sandwich plates with anti-symmetric angle-ply face sheets – Analytical evaluation of higher order refined computational models

K. Swaminathan *, S.S. Patil, M.S. Nataraja, K.S. Mahabaleswara

Department of Civil Engineering, National Institute of Technology Karnataka, Srinivasnagar, Karnataka 575 025, India

Abstract

The aim of the present study is to assess the accuracy of the few computational models based on various shear deformation theories in predicting the bending behaviour of sandwich plates with anti-symmetric angle-ply face sheets under static loading. Five two-dimensional models available in the literature are used for the present evaluation. The performance of the various models is evaluated on a simply supported laminated plate under sinusoidal loading. The equations of equilibrium are derived using the *principle of minimum potential energy* (PMPE). Analytical solution method using double Fourier series approach is used in conjunction with the admissible boundary conditions. The accuracy of each model is established by comparing the results of composite plates with the exact solutions already available in the literature. After establishing the correctness of the theoretical formulations and the solution method, benchmark results for transverse displacement, in-plane stresses, moment and shear stress resultants are presented for the multilayer sandwich plates.

Keywords: Analytical solutions; Flexural; Shear deformation; Higher-order theory; Sandwich plates

1. Introduction

Sandwich plates are basically a special form of fibre reinforced plates composed of two thin strong, stiff layers (face sheets) which resist bending bonded to a relatively thicker, less dense layer (core) to resist shear force or also made up of alternative arrangement of thin stiff layers and thick flexible cores. Because of their lightweight and high stiffness, sandwich plates and shells are being used in aerospace, shipbuilding and other industries. The face sheets are basically prepared from unidirectional fibre reinforced laminated composites. The core is a thick layer of a lower density material made up of foam polymer or honeycomb material. The methods of analysing sandwich structures and numerical solutions for the standard problems are well documented in the books by Allen [1] and Plantema [2]. For an extensive review of literature for the analysis of sandwich structures the reader may consult the articles by Habib [3,4], Bert and Francis [5] and Burton and Noor [6]. A selective review of the various analytical and numerical methods used for the stress analysis of laminated composite and sandwich plates was presented by Kant and Swaminathan [7]. Analytical formulations, solutions and comparison of numerical results for the buckling, free vibration, stress analyses of cross ply composite and sandwich plates based on the higher order refined theories already reported in the literature by Kant [8], Pandya and Kant [9–13] and Kant and Manjunatha [14] were presented recently by Kant and Swaminathan [15–18]. Recently, the theoretical formulations and solutions for the static analysis of anti-symmetric angle-ply laminated composite and sandwich plates using a nine degrees of freedom computational model were presented by Swaminathan and Ragounadin [19]. Even though a large number of publications exist on the modelling and analysis of sandwich structures using various two-dimensional displacement models, there is as

---

[*] Corresponding author. Tel.: +91 824 2474340; fax: +91 824 2474033.
E-mail address: swami@vasnet.co.in (K. Swaminathan).

0263-8223/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.compstruct.2006.04.007

such no quantitative assessment made using the various models. In this paper an attempt has been made to compare and assess quantitatively the accuracy of the results obtained using the various higher order models in predicting the static–flexural response of simply supported sandwich plate subjected to sinusoidal transverse load.

2. Displacement models

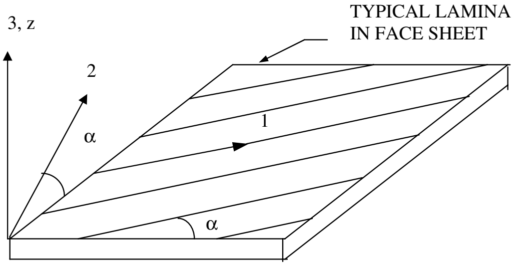
For the purpose of evaluation, the following higher order displacement models are considered. The geometry of the sandwich plate with positive set of the co-ordinate axes, physical mid-plane displacement terms is shown in Fig. 1.

Model-1 [14]

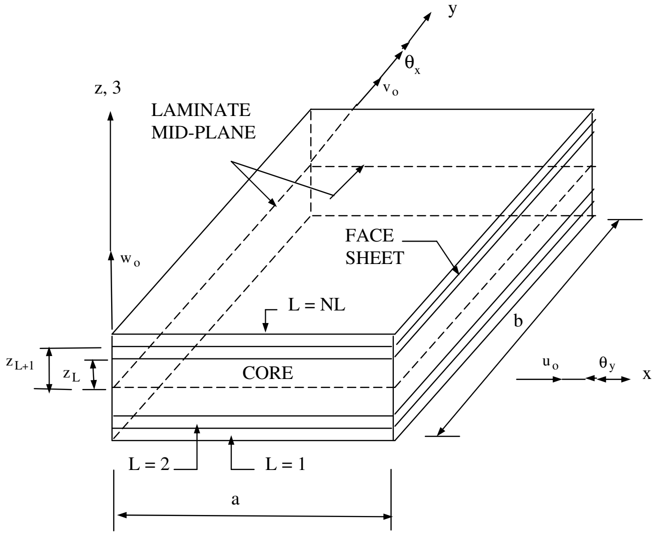
$$\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) + z\theta_z(x,y) + z^2 w_o^*(x,y) + z^3\theta_z^*(x,y) \end{aligned} \tag{1}$$

Model-2 [13]

$$\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) + z^2 u_o^*(x,y) + z^3\theta_x^*(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) + z^2 v_o^*(x,y) + z^3\theta_y^*(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned} \tag{2}$$



(1,2,3) - LAMINA REFERENCE AXES



(x,y,z) - LAMINATE REFERENCE AXES

Fig. 1. Geometry of a sandwich plate with positive set of lamina/laminate reference axes, displacement components and fibre orientation.

Though the above two models were already reported earlier in the literature and numerical results were presented using finite element formulations, analytical solutions for sandwich plates with angle-ply face sheets are obtained for the first time in this investigation and so the results obtained using the above two models are referred to as *present* in all the tables and figures. In addition to the above, the following higher order models and the first order model developed by other investigators are also considered for the evaluation. Analytical formulations developed and numerical results generated independently using these models are also being presented here with a view to have all the results on a common platform.

Model-3 [20]

$$\begin{aligned} u(x,y,z) &= u_o(x,y) + z\left[\theta_x(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left\{\theta_x(x,y) + \frac{\partial w_o}{\partial x}\right\}\right] \\ v(x,y,z) &= v_o(x,y) + z\left[\theta_y(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left\{\theta_y(x,y) + \frac{\partial w_o}{\partial y}\right\}\right] \\ w(x,y,z) &= w_o(x,y) \end{aligned} \tag{3}$$

Model-4 [21]

$$\begin{aligned} u(x,y,z) &= u_o(x,y) - z\frac{\partial w_o^{\mathrm{b}}}{\partial x} - \frac{4z^3}{3h^2}\frac{\partial w_o^{\mathrm{s}}}{\partial x} \\ v(x,y,z) &= v_o(x,y) - z\frac{\partial w_o^{\mathrm{b}}}{\partial y} - \frac{4z^3}{3h^2}\frac{\partial w_o^{\mathrm{s}}}{\partial y} \\ w(x,y,z) &= w_o^{\mathrm{b}}(x,y) + w_o^{\mathrm{s}}(x,y) \end{aligned} \tag{4}$$

Model-5 [22]

$$\begin{aligned} u(x,y,z) &= u_o(x,y) + z\theta_x(x,y) \\ v(x,y,z) &= v_o(x,y) + z\theta_y(x,y) \\ w(x,y,z) &= w_o(x,y) \end{aligned} \tag{5}$$

where the terms u, v and w are the displacements of a general point (x,y,z) in the laminate domain in the x, y and z directions, respectively. The parameters u_o, v_o are the in-plane displacements w_o, w_o^{b} and w_o^{s} are the transverse displacement, it's bending and shear components, respectively, of a point (x,y) on the middle plane. The functions θ_x, θ_y are rotations of the normal to the middle plane about y and x axes, respectively. The parameters u_o^*, v_o^*, w_o^*, θ_x^*, θ_y^*, θ_z^* and θ_z are the higher-order terms in the Taylor's series expansion and they represent higher-order transverse cross-sectional deformation modes.

3. Numerical results and discussions

To study the accuracy of prediction of the static–flexural response using the various higher order displacement models given in the preceding section, the numerical examples solved are described and discussed. For all the problems a simply supported plate subjected to sinusoidal load is considered for comparison. Results are obtained in closed-form using Navier's solution technique for the above geometry and loading. A shear correction factor of 5/6 is used to obtain the results using the displacement model based on the first order shear deformation theory.

The following sets of data were used in obtaining numerical results.

Material 1

$E_1 = 40 \times 10^6$ psi (276 GPa)

$E_2 = E_3 = 1 \times 10^6$ psi (6.895 GPa)

$G_{12} = G_{13} = 0.5 \times 10^6$ psi (3.45 GPa)

$G_{23} = 0.6 \times 10^6$ psi (4.12 GPa)

$\upsilon_{12} = \upsilon_{23} = \upsilon_{13} = 0.25$

Material 2

Face sheets (*Graphite Epoxy T300/934*)

$E_1 = 19 \times 10^6$ psi (131 GPa)

$E_2 = 1.5 \times 10^6$ psi (10.34 GPa)

$E_2 = E_3 \quad G_{12} = 1 \times 10^6$ psi (6.895 GPa)

$G_{13} = 0.90 \times 10^6$ psi (6.205 GPa)

$G_{23} = 1 \times 10^6$ psi (6.895 GPa)

$\upsilon_{12} = 0.22 \quad \upsilon_{13} = 0.22 \quad \upsilon_{23} = 0.49$

Core (*isotropic*)

$E_1 = E_2 = E_3 = 2G = 1000$ psi (6.90×10^{-3} GPa)

$G_{12} = G_{13} = G_{23} = 500$ psi (3.45×10^{-3} GPa)

$\upsilon_{12} = \upsilon_{13} = \upsilon_{23} = 0$

Table 1
Non-dimensionalized transverse deflection in a simply supported anti-symmetric angle-ply ($\theta/-\theta\ldots$) square laminate under sinusoidal transverse load

θ	a/h	Theory	$\bar{w}$	
			$n = 2$	$n = 4$
15°	4	Ren	1.4989	1.3050
		Model-1 (present)	1.4258	1.2608
		Model-2 (present)	1.4596	1.2869
		Model-3	1.3307	1.1903
		Model-4	1.0813	0.9580
		Model-5	1.4485	1.1982
	10	Ren	0.6476	0.4505
		Model-1 (present)	0.6296	0.4423
		Model-2 (present)	0.6374	0.4446
		Model-3	0.6213	0.4329
		Model-4	0.5672	0.3785
		Model-5	0.6361	0.4289
	100	Ren	0.4680	0.2668
		Model-1 (present)	0.4621	0.2662
		Model-2 (present)	0.4679	0.2667
		Model-3	0.4678	0.2666
		Model-4	0.4672	0.2660
		Model-5	0.4679	0.2666
30°	4	Ren	1.4865	1.0943
		Model-1 (present)	1.3439	1.0399
		Model-2 (present)	1.3775	1.0605
		Model-3	1.1082	0.9494
		Model-4	1.0609	0.8993
		Model-5	1.2464	0.9462
	10	Ren	0.6731	0.3543
		Model-1 (present)	0.6367	0.3439
		Model-2 (present)	0.6432	0.3454
		Model-3	0.5985	0.3291
		Model-4	0.5888	0.3182
		Model-5	0.6177	0.3244
	100	Ren	0.4975	0.2049
		Model-1 (present)	0.4931	0.2046
		Model-2 (present)	0.4972	0.2048
		Model-3	0.4967	0.2046
		Model-4	0.4966	0.2045
		Model-5	0.4969	0.2046

Table 2
Non-dimensionalized transverse deflection in a simply supported two layered anti-symmetric angle-ply ($\theta/-\theta$) rectangular ($b = 3a$) laminate under sinusoidal transverse load

θ	a/h	Theory	$\bar{w}$
30°	4	Ren	2.8881
		Model-1 (present)	2.6635
		Model-2 (present)	2.6980
		Model-3	2.3752
		Model-4	2.3709
		Model-5	2.6093
	10	Ren	1.5787
		Model-1 (present)	1.5321
		Model-2 (present)	1.5388
		Model-3	1.4872
		Model-4	1.4864
		Model-5	1.5212
	100	Ren	1.3163
		Model-1 (present)	1.3120
		Model-2 (present)	1.3158
		Model-3	1.3154
		Model-4	1.3154
		Model-5	1.3158
45°	4	Ren	3.9653
		Model-1 (present)	3.6239
		Model-2 (present)	3.6716
		Model-3	3.1562
		Model-4	3.0973
		Model-5	3.3816
	10	Ren	2.3953
		Model-1 (present)	2.3215
		Model-2 (present)	2.3323
		Model-3	2.2440
		Model-4	2.2326
		Model-5	2.2786
	100	Ren	2.0686
		Model-1 (present)	2.0609
		Model-2 (present)	2.0679
		Model-3	2.0673
		Model-4	2.0671
		Model-5	2.0677

Results reported in the tables are using the following non-dimensional form:

$$\bar{u} = u\left(\frac{100h^3E_2}{p_oa^4}\right) \quad \bar{v} = v\left(\frac{100h^3E_2}{p_oa^4}\right) \quad \bar{w} = w\left(\frac{100h^3E_2}{p_oa^4}\right)$$

$$\bar{\sigma}_x = \sigma_x\left(\frac{h^2}{p_oa^2}\right) \quad \bar{\sigma}_y = \sigma_y\left(\frac{h^2}{p_oa^2}\right) \quad \bar{\tau}_{xy} = \tau_{xy}\left(\frac{h^2}{p_oa^2}\right)$$

$$\overline{M}_x = M_x\left(\frac{1}{p_oa^2}\right) \quad \overline{M}_y = M_y\left(\frac{1}{p_oa^2}\right) \quad \overline{M}_{xy} = M_{xy}\left(\frac{1}{p_oa^2}\right)$$

$$\overline{Q}_x = Q_x\left(\frac{1}{p_oa}\right) \quad \overline{Q}_y = Q_y\left(\frac{1}{p_oa}\right)$$

Unless otherwise specified within the table(s) the locations (i.e. x-, y-, and z-coordinates) for maximum values of displacements and stresses for the present evaluations are as follows (see Tables 1 and 2):

In-plane displacement (u): $(0, b/2, \pm h/2)$
In-plane displacement (v): $(a/2, 0, \pm h/2)$
Transverse displacement (w): $(a/2, b/2, 0)$
In-plane normal stress (σ_x): $(a/2, b/2, \pm h/2)$
In-plane normal stress (σ_y): $(a/2, b/2, \pm h/2)$
In-plane shear stress (τ_{xy}): $(0, 0, \pm h/2)$
Bending stress resultant: M_x: $(a/2, b/2, 0)$ M_y: $(a/2, b/2, 0)$
M_{xy}: $(0, 0, 0)$
Shear stress resultant: Q_x: $(0, b/2, 0)$ Q_y: $(a/2, 0, 0)$

Example 1. In the case of a square and rectangular thick plates (a/h ratio 4 and 10) with different fibre orientations considered, the transverse displacement values predicted by model-2 is very much closer to the values reported by Ren [23]. All other models show large difference in displacement values. For a/h ratio equal to 4 and fibre orientation equal to 15°, the transverse deflection $\bar{w}$ values predicted by model-1, model-2, model-3, model-4 and model-5 are 4.88%, 2.62%, 11.22%, 27.86%, and 3.36% lower for a two layered square plate and 3.39%, 1.39%, 8.79%, 26.59% and 8.18% lower for a four layered square plate as compared to the values obtained by Ren. Both for the thin ($a/h = 100$) square and rectangular plates, all the models give almost the same results and they are in very good agreement with those given by Ren. The non-dimensionalized moment and shear stress resultants $\overline{M}_x$, $\overline{M}_y$, $\overline{M}_{xy}$, $\overline{Q}_x$ and $\overline{Q}_y$ of a two layered square composite plate for different a/h ratios and fibre orientations are given in Table 3. It can be observed that for all the range of parameters considered, the moment and shear stress resultants values predicted by model-1 and model-2 are almost same and very much closer to Ren values. The computed values of all other models show very large deviation and the models are unable to provide accurate estimates of bending and shear stress resultants particularly for thick plates.

Table 3
Non-dimensionalized stress resultants in a simply supported two layered anti-symmetric angle-ply ($\theta/-\theta$) square laminate under sinusoidal transverse load

θ	a/h	Theory	$\overline{M}_x$	$\overline{M}_y$	$\overline{M}_{xy}$	$\overline{Q}_x$	$\overline{Q}_y$
15°	4	Ren	0.0689	0.0150	−0.0087	–	–
		Model-1 (present)	0.0671	0.0155	−0.0092	0.2401	0.0781
		Model-2 (present)	0.0676	0.0145	−0.0096	0.2425	0.0758
		Model-3	0.0632	0.0132	−0.0124	0.2163	0.0754
		Model-4	0.0770	0.0093	−0.0075	0.1363	0.1596
		Model-5	0.0625	0.0138	−0.0125	0.2355	0.0828
	10	Ren	0.0761	0.0107	−0.0073	–	–
		Model-1 (present)	0.0756	0.0107	−0.0075	0.2611	0.0572
		Model-2 (present)	0.0756	0.0106	−0.0076	0.2613	0.0569
		Model-3	0.0746	0.0103	−0.0081	0.2474	0.0550
		Model-4	0.0776	0.0096	−0.0071	0.1398	0.1636
		Model-5	0.0746	0.0105	−0.0081	0.2599	0.0585
	100	Ren	0.0777	0.0096	−0.0070	–	–
		Model-1 (present)	0.0777	0.0096	−0.0069	0.2662	0.0520
		Model-2 (present)	0.0776	0.0096	−0.0070	0.2660	0.0522
		Model-3	0.0776	0.0096	−0.0070	0.2552	0.0496
		Model-4	0.0777	0.0096	−0.0070	0.1404	0.1644
		Model-5	0.0776	0.0096	−0.0070	0.2660	0.0523
30°	4	Ren	0.0472	0.0224	−0.0159	–	–
		Model-1 (present)	0.0456	0.0220	−0.0168	0.1961	0.1221
		Model-2 (present)	0.0453	0.0212	−0.0174	0.1971	0.1212
		Model-3	0.0411	0.0184	−0.0209	0.1719	0.1101
		Model-4	0.0467	0.0183	−0.0182	0.1351	0.1479
		Model-5	0.0410	0.0189	−0.0207	0.1939	0.1245
	10	Ren	0.0477	0.0196	−0.0170	–	–
		Model-1 (present)	0.0473	0.0194	−0.0173	0.2029	0.1154
		Model-2 (present)	0.0471	0.0192	−0.0175	0.2029	0.1154
		Model-3	0.0462	0.0187	−0.0182	0.1851	0.1059
		Model-4	0.0434	0.0135	−0.0173	0.1295	0.0945
		Model-5	0.0462	0.0188	−0.0182	0.2022	0.1161
	100	Ren	0.0475	0.0188	−0.0175	–	–
		Model-1 (present)	0.0475	0.0188	−0.0174	0.2043	0.1139
		Model-2 (present)	0.0474	0.0188	−0.0175	0.2042	0.1141
		Model-3	0.0474	0.0188	−0.0176	0.1881	0.1048
		Model-4	0.0474	0.0188	−0.0176	0.1398	0.1531
		Model-5	0.0474	0.0188	−0.0176	0.2042	0.1141

Example 2. In order to study the flexural behaviour of laminated sandwich plate, a five layered square plate (30°/−30°/core/30°/−30°) with isotropic core and anti-symmetric angle-ply face sheets is considered. Material set 2 is used. The ratio of the thickness of core to thickness of the face sheet t_c/t_f considered equal to 4. The non-dimensionalized

maximum values of transverse displacement $\bar{w}$, in-plane stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and $\bar{\tau}_{xy}$ for various values of side-to-thickness ratio are given in Table 4. In the case of thick plates with a/h ratio equal to 2, 4 and 10, the $\bar{w}$, $\bar{\sigma}_x$, $\bar{\sigma}_y$ and $\bar{\tau}_{xy}$ values predicted by model-1 and model-2 are very much closer whereas model-3, model-4 and model-5 very much underpredicts these values. For a thick plate with a/h ratio equal to 2, the values of $\bar{w}$ predicted by model-2, model-3, model-4 and model-5 are, respectively, 4.68% higher, 26.41%, 26.99% and 78.75% lower as compared to model-1. The difference between the models tends to reduce for thin and relatively thin plates. The through the thickness variation of in-plane displacements $\bar{u}$ and $\bar{v}$ for a plate with a/h ratio equal to 4 and ratio of the thickness of core to thickness of the face sheet t_c/t_f equal to 4 are shown in Figs. 2 and 3. It clearly indicates that the model-1 and the model-2 predict the realistic through the thickness variation of displacements more accurately than model-3, model-4 and model-5.

Example 3. A simply supported five layered square sandwich plate (30°/−30°/core/30°/−30°) with isotropic core

Table 4
Non-dimensionalized transverse deflection and in-plane stresses in a simply supported five layered anti-symmetric angle-ply (30°/−30°/core/30°/−30°) square sandwich plate under sinusoidal transverse load

a/h	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
2	Model-1 (present)	38.0751	2.6289	1.2761	−1.2922
	Model-2 (present)	39.8563	2.6125	1.1894	−1.3466
	Model-3	28.0172	1.3878	0.6746	−0.7949
	Model-4	27.7984	1.5070	0.6417	−0.7405
	Model-5	8.0923	0.2196	0.1648	−0.2181
4	Model-1 (present)	13.0335	1.0175	0.5013	−0.5859
	Model-2 (present)	13.2429	1.0108	0.4982	−0.5932
	Model-3	8.1972	0.5576	0.3002	−0.3694
	Model-4	8.0144	0.6495	0.2744	−0.3269
	Model-5	2.5977	0.2803	0.1510	−0.1948
10	Model-1 (present)	2.9394	0.4219	0.2050	−0.2622
	Model-2 (present)	2.9521	0.4241	0.2102	−0.2618
	Model-3	1.9692	0.3531	0.1699	−0.2139
	Model-4	1.9047	0.3847	0.1610	−0.1992
	Model-5	1.0182	0.3222	0.1415	−0.1787
20	Model-1 (present)	1.2839	0.3524	0.1567	−0.1991
	Model-2 (present)	1.2868	0.3527	0.1581	−0.1984
	Model-3	1.0300	0.3367	0.1471	−0.1850
	Model-4	1.0107	0.3462	0.1445	−0.1806
	Model-5	0.7884	0.3304	0.1396	−0.1755
50	Model-1 (present)	0.8032	0.3368	0.1422	−0.1791
	Model-2 (present)	0.8044	0.3362	0.1421	−0.1784
	Model-3	0.7627	0.3338	0.1403	−0.1762
	Model-4	0.7594	0.3354	0.1398	−0.1754
	Model-5	0.7237	0.3328	0.1391	−0.1746
100	Model-1 (present)	0.7339	0.3348	0.1401	−0.1761
	Model-2 (present)	0.7350	0.3340	0.1398	−0.1754
	Model-3	0.7246	0.3335	0.1393	−0.1749
	Model-4	0.7238	0.3333	0.1392	−0.1747
	Model-5	0.7149	0.3332	0.1390	−0.1745

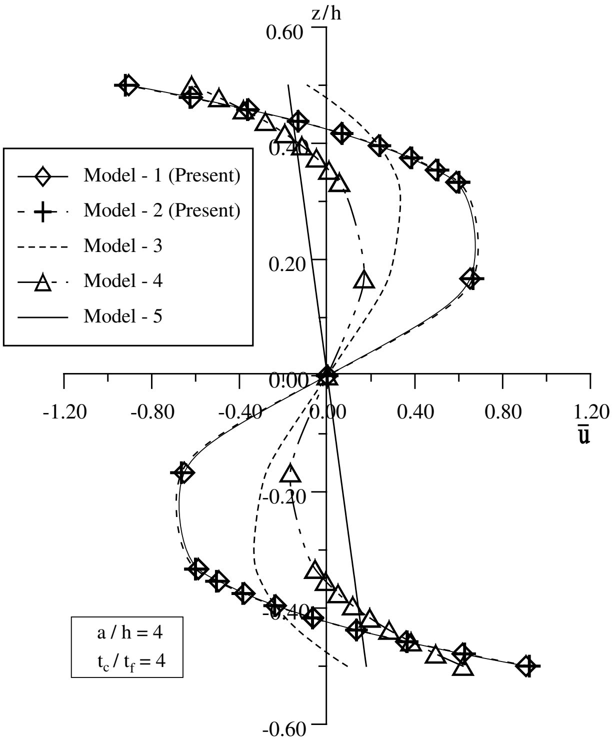


Fig. 2. Variation of non-dimensionalized in-plane displacement ($\bar{u}$) through the thickness (z/h) of a five layered (30°/−30°/core/30°/−30°) simply supported anti-symmetric angle-ply square sandwich plate under sinusoidal transverse load.

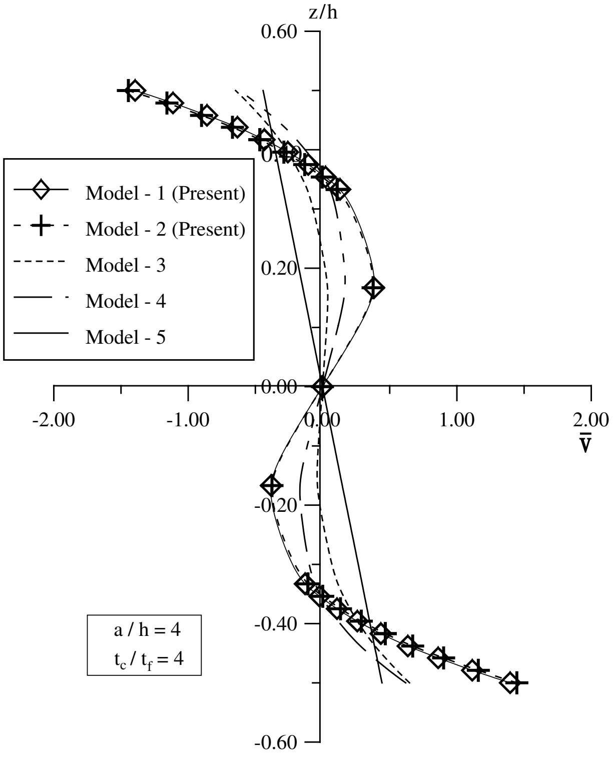


Fig. 3. Variation of non-dimensionalized in-plane displacement ($\bar{v}$) through the thickness (z/h) of a five layered (30°/−30°/core/30°/−30°) simply supported anti-symmetric angle-ply square sandwich plate under sinusoidal transverse load.

and anti-symmetric angle-ply face sheets is considered. Material set 2 is used. The side-to-thickness a/h ratio considered equal to 4. The non-dimensionalized maximum values of transverse displacement $\bar{w}$, in-plane stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and $\bar{\tau}_{xy}$ for various values thickness of core to thickness of the face sheet t_c/t_f ratio are given in Table 5. For all the values of t_c/t_f ratio the displacement and stress values obtained using model-1 and model-2 are in good agreement whereas considerable difference exists between these two and other models. In particular the first order theory (model-5) very much underpredicts these values. For plates with t_c/t_f equal to 10 the value of $\bar{w}$ predicted by model-2, model-3, model-4 and model-5 are, respectively, 1.63% higher, 53.30%, 53.73% and 94.89% lower as compared to model-1. Similar pattern in the percentage difference can be observed in the transverse displacement and in-plane stress values for all other ratios of t_c/t_f.

Example 4. A simply supported five layered square sandwich plate ($\theta/-\theta/\text{core}/\theta/-\theta$) with isotropic core and anti-symmetric angle-ply face sheets with the ratio of the thickness of core to thickness of the face sheet t_c/t_f equal to 4 is considered. Material set 2 is used. The side-to-thickness a/h ratio considered equal to 4. The non-dimensionalized maximum values of transverse displacement $\bar{w}$, in-plane stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$ and $\bar{\tau}_{xy}$ for various values of fibre orientation θ are given in Table 6. For all the values of θ considered the results computed using model-1 and model-2 are very much closer but a considerable difference exists between these two and other models. For a plate with θ equal to 15° the value of $\bar{w}$ predicted by model-2, model-3, model-4 and model-5 are, respectively, 1.49% higher, 34.69%, 39.76% and 77.31% lower as compared to model-1. A similar observation can be made for in-plane stresses $\bar{\sigma}_x$, $\bar{\sigma}_y$, $\bar{\tau}_{xy}$ and transverse displacement for any given value of fibre orientation.

Table 5
Non-dimensionalized transverse deflection and in-plane stresses in a simply supported five layered anti-symmetric angle-ply (30°/−30°/core/30°/−30°) square sandwich plate under sinusoidal transverse load

t_c/t_f	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
4	Model-1 (present)	13.0335	1.0175	0.5013	−0.5859
	Model-2 (present)	13.2429	1.0108	0.4982	−0.5932
	Model-3	8.1972	0.5576	0.3002	−0.3694
	Model-4	8.0144	0.6495	0.2744	−0.3269
	Model-5	2.5977	0.2803	0.1510	−0.1948
10	Model-1 (present)	96.6657	3.4494	1.6989	−1.8110
	Model-2 (present)	98.2424	3.2296	1.5387	−1.8137
	Model-3	45.1407	1.4128	0.7554	−0.9078
	Model-4	44.7308	1.6416	0.6980	−0.8102
	Model-5	4.9418	0.4682	0.2591	−0.3388
20	Model-1 (present)	379.1888	7.1995	3.5922	−3.6643
	Model-2 (present)	383.1255	6.5706	3.1104	−3.6402
	Model-3	195.8631	3.3148	1.7080	−2.0122
	Model-4	195.1197	3.7562	1.6034	−1.8313
	Model-5	8.8633	0.7941	0.4445	−0.5846
50	Model-1 (present)	744.3126	6.6225	3.6918	−3.9285
	Model-2 (present)	745.8548	6.0370	3.2452	−3.9325
	Model-3	647.4147	5.2918	2.9203	−3.5121
	Model-4	645.7906	6.3147	2.6877	−3.1055
	Model-5	20.5859	1.7850	1.0053	−1.3272
100	Model-1 (present)	795.7270	5.3705	3.6670	−4.2115
	Model-2 (present)	796.6311	4.9925	3.3528	−4.3103
	Model-3	783.8121	4.9550	3.3335	−4.2592
	Model-4	780.8080	6.8890	2.9009	−3.5001
	Model-5	39.9039	3.4519	1.9429	−2.5680

Table 6
Non-dimensionalized transverse deflection and in-plane stresses in a simply supported five layered anti-symmetric angle-ply ($\theta/-\theta/\text{core}/\theta/-\theta$) square sandwich plate under sinusoidal transverse load

θ	Theory	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\sigma}_y$	$\bar{\tau}_{xy}$
15°	Model-1 (present)	13.5481	1.4147	0.3491	−0.3906
	Model-2 (present)	13.7509	1.4230	0.3524	−0.4000
	Model-3	8.8488	0.7862	0.2407	−0.2787
	Model-4	8.1608	0.9648	0.1633	−0.1962
	Model-5	3.0735	0.3958	0.1296	−0.1564
30°	Model-1 (present)	13.0335	1.0175	0.5013	−0.5859
	Model-2 (present)	13.2429	1.0108	0.4982	−0.5932
	Model-3	8.1972	0.5576	0.3002	−0.3694
	Model-4	8.0144	0.6495	0.2744	−0.3269
	Model-5	2.5977	0.2803	0.1510	−0.1948
45°	Model-1 (present)	12.7676	0.7313	0.7313	−0.6698
	Model-2 (present)	12.9778	0.7270	0.7270	−0.6705
	Model-3	7.9482	0.4237	0.4237	−0.3908
	Model-4	7.9482	0.4237	0.4237	−0.3908
	Model-5	2.4409	0.2113	0.2113	−0.1973
60°	Model-1 (present)	13.0334	0.5013	1.0174	−0.5859
	Model-2 (present)	13.2429	0.4982	1.0108	−0.5932
	Model-3	8.1972	0.3002	0.5576	−0.3694
	Model-4	8.0144	0.2744	0.6495	−0.3269
	Model-5	2.5977	0.1510	0.2803	−0.1948
75°	Model-1 (present)	13.5481	0.3491	1.4147	−0.3906
	Model-2 (present)	13.7509	0.3524	1.4230	−0.4000
	Model-3	8.8488	0.2407	0.7862	−0.2787
	Model-4	8.1608	0.1633	0.9648	−0.1962
	Model-5	3.0735	0.1296	0.3958	−0.1564

Example 5. A simply supported five layered square sandwich plate (30°/−30°/core/30°/−30°) with isotropic core and anti-symmetric angle-ply face sheets are considered. Material set 2 is used. The ratio of the thickness of core to thickness of the face sheet t_c/t_f considered equal to 4. The non-dimensionalized moment and shear stress resultants $\overline{M}_x$, $\overline{M}_y$, $\overline{M}_{xy}$, $\overline{Q}_x$ and $\overline{Q}_y$ fr varying slenderness ratios and for a given ratio of the thickness of the core to the thickness of the face sheet t_c/t_f equal to 4 are compared in Table 7. In the case of a thick plate considerable deviation exists in the moment stress resultant values predicted by other models compared to model-1. The deviation is much higher in the case of model-4. For thin plates all the models render the same results. For all the values of a/h ratios considered, reasonably good agreement exists

Table 7
Non-dimensionalized stress resultants in a simply supported five layered anti-symmetric angle-ply (30°/−30°/core/30°/−30°) square sandwich plate under sinusoidal transverse load

a/h	Theory	$\overline{M}_x$	$\overline{M}_y$	$\overline{M}_{xy}$	$\overline{Q}_x$	$\overline{Q}_y$
2	Model-1 (present)	0.0322	0.0261	−0.0214	0.1687	0.1495
	Model-2 (present)	0.0277	0.0212	−0.0261	0.1693	0.1489
	Model-3	0.0243	0.0212	−0.0278	0.2843	0.2800
	Model-4	0.0414	0.0174	−0.0211	0.2750	0.2898
	Model-5	0.0274	0.0199	−0.0269	0.1709	0.1473
4	Model-1 (present)	0.0276	0.0220	−0.0258	0.1679	0.1503
	Model-2 (present)	0.0265	0.0206	−0.0271	0.1684	0.1498
	Model-3	0.0278	0.0200	−0.0267	0.3262	0.2825
	Model-4	0.0410	0.0171	−0.0215	0.2966	0.3125
	Model-5	0.0346	0.0183	−0.0241	0.1847	0.1335
10	Model-1 (present)	0.0340	0.0188	−0.0241	0.1830	0.1352
	Model-2 (present)	0.0340	0.0186	−0.0243	0.1833	0.1349
	Model-3	0.0363	0.0180	−0.0234	0.3674	0.2550
	Model-4	0.0409	0.0170	−0.0216	0.3031	0.3194
	Model-5	0.0396	0.0172	−0.0222	0.1942	0.1240
20	Model-1 (present)	0.0385	0.0175	−0.0225	0.1921	0.1262
	Model-2 (present)	0.0386	0.0175	−0.0225	0.1922	0.1260
	Model-3	0.0395	0.0173	−0.0222	0.3808	0.2436
	Model-4	0.0409	0.0170	−0.0216	0.3041	0.3204
	Model-5	0.0405	0.0170	−0.0218	0.1960	0.1222
50	Model-1 (present)	0.0404	0.0170	−0.0218	0.1959	0.1223
	Model-2 (present)	0.0405	0.0170	−0.0218	0.1959	0.1223
	Model-3	0.0407	0.0170	−0.0217	0.3856	0.2397
	Model-4	0.0409	0.0169	−0.0216	0.3044	0.3209
	Model-5	0.0408	0.0170	−0.0217	0.1966	0.1216
100	Model-1 (present)	0.0407	0.0170	−0.0217	0.1965	0.1217
	Model-2 (present)	0.0408	0.0170	−0.0217	0.1965	0.1217
	Model-3	0.0408	0.0170	−0.0217	0.3862	0.2390
	Model-4	0.0409	0.0169	−0.0216	0.3044	0.3208
	Model-5	0.0409	0.0169	−0.0217	0.1967	0.1215

in the shear stress resultants values obtained using model-1 and model-2 whereas the results of model-3 and model-4 deviate somewhat more as compared to the deviation in values shown by model-5.

4. Conclusion

Analytical solutions to the static analysis of simply supported anti-symmetric angle-ply composite and sandwich plates are presented. Comparative study on the static–flexural response of various shear deformation theories applied to multilayer sandwich plates is done. Exact solutions already available in the literature are used for comparison. The results of all the models compared include the transverse displacement, the in-plane stresses, moment and shear stress resultants. From the extensive numerical results presented in this paper it is concluded that both for the composite and sandwich plates, model-1 and model-2 considered in the present investigation predict the displacements, in-plane stresses and the stress resultants values with reasonable accuracy compared to other models.

References

[1] Allen HG. Analysis and design of structural sandwich panels. Oxford, UK: Pergamon Press; 1969.

[2] Plantema FJ. Sandwich construction, the bending and buckling of sandwich beams, plates and shells. New York: Wiley; 1966.

[3] Habib LM. A review of recent Russian work on sandwich construction. Int J Mech Sci 1964;6(6):483–7.

[4] Habib LM. A survey of modern developments in the analysis of sandwich structures. Appl Mech Rev 1965;18(2):93–8.

[5] Bert CW, Francis PH. Composite material mechanics: structural mechanics. AIAA J 1974;12(9):1173–86.

[6] Burton WS, Noor AK. Assessment of computational models for sandwich panels and shells. Comput Meth Appl Mech Eng 1995; 124(1–2):125–51.

[7] Kant T, Swaminathan K. Estimation of transverse/interlaminar stresses in laminated composites – a selective review and survey of current developments. Compos Struct 2001;49(1):65–75.

[8] Kant T. Numerical analysis of thick plates. Comput Meth Appl Mech Eng 1982;31:1–18.

[9] Pandya BN, Kant T. A consistent refined theory for flexure of a symmetric laminate. Mech Res Commun 1987;14:107–13.

[10] Pandya BN, Kant T. Higher order shear deformable theories for flexure of sandwich plates – finite element evaluations. Int J Solids Struct 1988;24(12):1267–86.

[11] Pandya BN, Kant T. Flexure analysis of laminated composites using refined higher order C° plate bending elements. Comput Meth Appl Mech Eng 1988;66:173–98.

[12] Pandya BN, Kant T. A refined higher order generally orthotropic C° plate bending element. Comput Struct 1988;28:119–33.

[13] Pandya BN, Kant T. Finite element stress analysis of laminated composites using higher order displacement model. Compos Sci Technol 1988;32:137–55.

[14] Kant T, Manjunatha BS. An unsymmetric FRC laminate C° finite element model with 12 degrees of freedom per node. Eng Comput 1988;5(3):300–8.

[15] Kant T, Swaminathan K. Analytical solutions using a higher order refined theory for the stability analysis of laminated composite and sandwich plates. Struct Eng Mech, An Int J 2000;10(4):337–57.

[16] Kant T, Swaminathan K. Free vibration of isotropic, orthotropic and multilayer plates based on higher order refined theories. J Sound Vibr 2001;241(2):319–27.

[17] Kant T, Swaminathan K. Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher order refined theory. Compos Struct 2001;53(1):73–85.

[18] Kant T, Swaminathan K. Analytical solutions for static analysis of laminated composite and sandwich plates based on a higher order refined theory. Compos Struct 2002;56(4):329–44.

[19] Swaminathan K, Ragounadin D. Analytical solutions using a higher order refined theory for the static analysis of antisymmetric angle ply composite and sandwich plates. Compos Struct 2004;64(3–4):405–17.

[20] Reddy JN. A simple higher-order theory for laminated composite plates. J Appl Mech 1984;51:745–52.

[21] Senthilnathan NR, Lim KH, Lee KH, Chow ST. Buckling of shear-deformable plates. AIAA J 1987;25(9):1268–71.

[22] Whitney JM, Pagano NJ. Shear deformation in heterogeneous anisotropic plates. J Appl Mech 1970;37(4):1031–6.

[23] Ren JG. Bending, vibration and buckling of laminated plates. In: Cheremisinoff NP, editor. Handbook of ceramics and composites, vol. 1. New York: Marcel Dekker; 1990. p. 413–50.