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Channel blind identification based on cyclostationarity and group delay

S.V. Narasimhan[a,*], M. Hazarathaiah[b,1], P.V.S. Giridhar[b,2]

[a]*Digital Signal Processing Systems Lab., Aerospace Electronics Division, National Aerospace Laboratory, Bangalore-560017, India*
[b]*Department of Electronics and Communication Engineering, National Institute of Technology, Karnataka, Surathkal-574157, India*

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Abstract

A new approach for channel blind identification based on second order cyclostationary statistics and the group delay has been proposed. In this, two methods are proposed. In both the methods, the correction is applied to the basic phase estimate *for both the poles and zeros*, *in the group delay domain*. The basic phase estimate is derived from the spectral correlation density (SCD) of the system output. In the first method, the phase correction is based on magnitude group delay. In the second method, not only the phase correction but also an *improved system magnitude estimate* of better variance and frequency resolution is derived based on modified magnitude group delay. The results indicate a significant improvement in performance for both the methods. For the first method in the absence of noise, the percentage normalized mean square error is reduced by about 85% over that of the existing non-parametric method. The second method in the presence of noise (SNR = 5 dB), provides a reduction of 74% over the existing non-parametric method and 57% over the existing combined parametric and non-parametric methods.

Keywords: Cyclostationarity; Group delay functions and channel blind-identification

1. Introduction

Communication channels, acoustic paths and vocal tract etc., are of non-minimum phase nature and their actual identification is essential. However, the autocorrelation-based system identification is limited to minimum phase systems. Bispectrum, the higher order statistics, has been used to extract the complete system phase [1,2]. However, they cannot handle channel inputs which are symmetrical or near symmetrical amplitude distributions. Further, they are very sensitive to timing jitters and may involve large amount of data for ensemble averaging in estimating higher

*Corresponding author. Tel.: +80 527 3351; fax: +80 527 0670.
E-mail address: narasim@css.cmmacs.ernet.in (S.V. Narasimhan).
[1]Presently, at Analog Devices (India) Ltd, Bangalore, India.
[2]Presently, at SASKEN, Bangalore, India.

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order statistics, processing and non-linear optimization [8].

Signals from the field of communication, rotating machinery, astronomy and weather exhibit cyclostationarity. Their spectral characteristics change periodically with time [3–5] and exhibit spectral correlation between frequency components. Methods based on second order cyclic statistics viz., spectral correlation density (SCD) are attractive as they require relatively less data and only second order statistics. They are also applicable to all types of distributions and are insensitive to stationary additive noise. In cyclostationarity-based methods, the channel identification is done by fractional sampling of its output and computing the complex cepstrum of the output cyclic autocorrelation [6]. The drawback of this is that it poses problems when the zeros are very close to unit circle. This drawback has been overcome in the absence of noise, by a subspace approach by introducing cyclostationarity at the transmitter [7]. In the parametric approach, the poles and zeros of a mixed phase ARMA system are identified using the SCD of its output and this requires information about the order of the system in advance.

In the non-parametric approach (NP) [6,8], at frequencies $(2\pi/p)$, p being the over-sampling rate, the phase information either cannot be computed or it will be in error. To reduce the phase estimation errors at the poles, a hybrid method [8] that uses the predetermined pole information by parametric method (NPP) has been reported. The parametric methods are in general applicable when the signal to noise ratio (SNR) is high and the assumed model matches the process under consideration.

The group delay (GD) of the Fourier transform (FT) phase has been used for system blind identification [9]. For the phase derived from SCD, the correction for the poles is applied in the GD domain using the information in the GD derived from the magnitude, the magnitude GD (MGD). This GD domain phase correction is simple and it performs better even at nominal SNR. However, it requires a good spectral smoothing for the SCD and this may reduce the frequency resolution for the phase.

Recently, a modified magnitude group delay (MGDM) [10] that reduces the variance of the spectral estimate without compromising on the frequency resolution has been proposed. The MGDM reduces the effect of the zeros close to the unit circle due to input driving or associated noise or due to signal truncation, without disturbing the system roots and hence reduces variance without affecting the frequency resolution.

This paper proposes new two-channel/system blind identification methods based on cyclostationarity and group delay functions. This is achieved by applying correction to the phase derived from the SCD of the output and the *correction is applied for both poles and zeros.* For the first method, the information in MGD is used for correcting the phase. For the second method, *the information in MGDM is not only used for correcting the phase but also in deriving an improved system magnitude estimate*. Their performance is significantly improved over the existing methods [8] and the second method is effective even at low SNR.

In this study, the performance comparison of the proposed method is limited only to those of SCD-based methods considered in [8] and not to the other SCD-based methods.

2. Blind identification based on cyclostationarity

This section briefly reviews the channel identification based on the cyclostationarity induced by oversampling the channel output and its limitations.

2.1. Cyclostationarity of the over-sampled channel output [8]

For the system identification, Fig. 1 shows a simplified base-band representation of the pulse

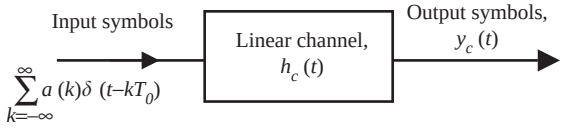

Fig. 1. Base-band representation of PAM data communication system.

amplitude modulation (PAM) communication system. Here, $h_c(t)$ is the composite channel impulse response that includes all interconnections between the symbol generator and output of the receiving filter. The channel is assumed to be linear, time-invariant, causal and bounded input-bounded output stable. The symbol source generates a zero mean unit variance i.i.d. (independent identically distributed) sequence $\{a_k\}$, each element of which comes from a constellation of PAM symbols. The received continuous time signal is given by

$$y_c(t) = \sum a(l)h_c(t - lT_0) + w(t), \tag{2.1}$$

T_0^{-1} is the baud rate (the rate at which signal level is changed and this depends upon the nature of the format used to represent the digital data) and $w(t)$ is the additive noise with noise spectral level N_0, independent of a_k. For a sampling period of T_s, $h(n) = h_c(t)|_{t=nT_s}$ and $w(n) = w(t)|_{t=nT_s}$, the discrete-time model is

$$y(n) = y_c(t)|_{t=nT_s} = \sum_l a(l)h(n - lP) + w(n),$$
$$P = T_0/T_s. \tag{2.2}$$

The cyclostationarity is induced at the receiver by fractional sampling, i.e., $P>1$ and is achieved by over sampling the system output with respect to baud rate. This results in a discrete cyclostationary process $y(n)$. The correlation function of $y(n)$ is

$$R_y(n+m,n) = E\{y(n+m)y^*(n)\}, \tag{2.3}$$

where $E\{.\}$ is the expectation operation. The cyclic correlation function of $y(n)$ [3] is

$$R_y^{(l\beta)}(m) = \sum_{n=0}^{P-1} R_y(n+m,n)e^{-jnl\beta},$$
$$\beta = 2\pi/P, l: \text{integer}. \tag{2.4}$$

The spectral correlation density (SCD) function is

$$S_y^{(l\beta)}(e^{j\omega}) = \sum_n R_y^{(l\beta)}(n)e^{-jn\omega}, \tag{2.5}$$

$$S_y^{(l\beta)}(e^{j\omega}) = E\{|a_n|^2\}H(e^{j\omega})H^*(e^{j(\omega-l\beta)}) + PN_o\delta(l), \tag{2.6}$$

which is achieved by using Eqs. (2.2), (2.3), and (2.4) in Eq. (2.5) [8]. In the present study, this equation forms the basis for channel identification and when $l\neq 0$, the SCD provides noise rejection.

2.2. Channel identification from its output SCD [8]

The objective is to identify the channel frequency response $H(e^{j\omega})$ from the SCD of the channel output $y(n)$ for a zero mean i.i.d. input.

2.2.1. Magnitude estimation

The system magnitude can be derived from Eq. (2.6) for both the cases viz., no noise and with noise.

For the case, when the output of a system is not corrupted by the stationary additive noise, the system magnitude is obtained by putting the value of $l = 0$ in Eq. (2.6) and is given by

$$|H(e^{j\omega})| = \frac{1}{\sigma}[S_y^{(0)}(\omega)]^{1/2}, \quad \sigma^2 = E\{|a_n|^2\}. \tag{2.7}$$

For the noisy or low SNR case, the system magnitude is derived by choosing the value of $l = 1$ in Eq. (2.6) and is given by

$$S_y^{(1)}(e^{j\omega}) = \sigma^2 H(e^{j\omega})H^*(e^{j(\omega-2\pi/P)}).$$

If the channel has no zeros on the unit circle then the system magnitude can be derived from $|S_y^{\beta}(e^{j\omega})|$ and is given by [8]

$$|H(e^{j\omega})| = \exp[\mathrm{DFT}(c_h(n))],$$
$$c_h(n) = \frac{c_1(n) - \delta(n)\ln(\sigma^2)}{1 + e^{j2n\pi/P}}, \tag{2.8a, b}$$

and

$$c_1(n) = \mathrm{IDFT}[\ln|S_y^{\beta}(e^{j\omega})|], \tag{2.9a}$$

where DFT is discrete FT, IDFT the inverse DFT. $c_1(n)$ is the real cepstrum of, $S_y^{\beta}[e^{j\omega}]$

$$c_h(n) = \mathrm{IDFT}[\ln|H(e^{j\omega})|], \tag{2.9b}$$

$c_h(n)$ is the cepstrum of $H[e^{j\omega}]$. However, for $2n/P = 2k + 1$ or $n = [k + 1/2]P$, k an integer; there is no solution. Thus as long as P is an odd integer, the magnitude of $|H(e^{j\omega})|$ can be completely identified from the SCD, even when a strong stationary noise is present but requires a relatively large number of data samples. ($\sigma^2 = 1$ as input with unit variance is assumed, otherwise also $H[e^{j\omega}]$ will be available within a scale factor).

2.2.2. Phase estimation

With $1 \neq 0$, Eq. (2.6) provides phase identification. If $\psi_l(e^{j\omega})$ and $\Phi(e^{j\omega})$ are the phase responses of the SCD $S_y^{l\beta}(e^{j\omega})$ and $H(e^{j\omega})$, respectively, then the output SCD phase and system phase are related by [8]

$$\phi(\tau) = \psi_l(\tau)/[1 - \exp\{(-j2\pi l\tau)/P\}], \tag{2.10}$$

$\psi_l(\tau)$ and $\phi(\tau)$ are their Cepstra with $\psi_l(\tau) \leftrightarrow \Psi_l(e^{j\omega})$ and $\phi(\tau) \leftrightarrow \Phi(e^{j\omega})$, respectively. However, *when*

$$1 - \exp\{(-j2\pi l\tau)/P\} = 0 \quad \text{or} \tau = mP/l, \quad m : \text{integer}, \tag{2.11}$$

cepstral phase information cannot be extracted from Eq. (2.10). Therefore, any arbitrary channel cannot be *completely identified from the cyclostationary statistics alone*. Further, this is supported by the fact that the discrete channels with a certain number of zeros uniformly placed in a circle cannot be identified properly [8]. Further, Eq. (2.6), will not extract any additional phase information from the SCD by setting $l = \pm 2, \pm 3, \ldots$. Therefore only the SCD evaluated at $l = 1$ is used. *Thus, the phase obtained from the incomplete set of cepstral coefficients will be highly inaccurate and hence needs correction*. This blind identification approach will be referred to as *nonparametric method* (NP).

For the case of a system with poles located at uniform spacing along a circle, it has been shown that the phase correction for poles [8], improves the identification significantly compared to that obtained by the NP method.

The inaccurate phase obtained from the incomplete set of cepstral coefficients by the NP method will be *corrected both for poles and zeros* and also an *improved system magnitude even in the presence of an observation noise*, will be derived by the proposed group delay approach.

3. Background

In this section, group delay functions which will be used in correcting the phase derived from the SCD and in obtaining an improved spectral magnitude in identifying the channel will be considered.

3.1. Group delay function (GD) [11]

For a mixed phase signal $x(n)$ which can also be the output of a system, the spectral magnitude and phase of $X(e^{j\omega})$ *are not related* by same cepstral coefficients and are given by [11],

$$\ln|X(e^{j\omega})| = \sum_{n=0}^{\infty} u(n)\cos(\omega n) \quad \text{and}$$

$$\theta(e^{j\omega}) = -\sum_{n=1}^{\infty} v(n)\sin(\omega n), \tag{3.1a, b}$$

$u(n)$ and $v(n)$ are cepstral coefficient sequences of the minimum phase equivalent systems derived from the spectral magnitude and phase, respectively. The GD negative derivative of the phase $\theta(e^{j\omega})$ called the phase GD (PGD), $\tau_p(e^{j\omega})$ is

$$\tau_p(e^{j\omega}) = -d\theta(\omega)/d\omega = \sum_{n=1}^{\infty} nv(n)\cos(\omega n), \tag{3.2a}$$

$\tau_p(e^{j\omega})$ can also be computed directly from $X(e^{j\omega})$ and $Y(e^{j\omega})$ which are the Fourier transforms $x(n)$ and from $y(n)$ by the relation

$$\tau_p(e^{j\omega}) = \frac{X_R(e^{j\omega})Y_R(e^{j\omega}) + X_I(e^{j\omega})Y_I(e^{j\omega})}{|X(e^{j\omega})|^2}, \tag{3.2b}$$

where $y(n) = nx(n)$ and R and I are the real and imaginary parts. If the phase $\theta(\omega)$ is known, $x(n)$ is the allpass sequence and $X(\omega) = e^{j\theta(\omega)}$.

If $v(n) = u(n) = c(n)$, the phase derived using Eq. (3.1) from magnitude corresponds to the phase of a minimum phase equivalent of the signal or system and the corresponding GD is called the magnitude GD (MGD), $\tau_m(e^{j\omega})$ is given by

$$\tau_m(e^{j\omega}) = \sum_{n=1}^{\infty} nc(n)\cos(\omega n). \tag{3.3}$$

For a minimum phase signal, the magnitude and phase are related by the same cepstral coefficients $c(n)$ (Eqs. (3.1a,b)) and $\tau_p(e^{j\omega}) = \tau_m(e^{j\omega})$. For a maximum phase signal, $u(n) = -v(n)$ [11], hence, $\tau_p(e^{j\omega}) = -\tau_m(e^{j\omega})$.

The significant values of MGD for a real pole are close to the origin and for a second order pole and are around the resonance frequency [12]. The MGD for the zeros is same as those of poles but opposite in sign. The MGD for a real and a complex pole is positive, but for real and complex zero is negative. If $[\tau_m(e^{j\omega})]^+$ and $[\tau_m(e^{j\omega})]^-$ are the positive and negative parts of the MGD, respectively, then the cepstral coefficients for the pole part, $u^+(n)$ and for the zero part, $u^-(n)$ are [12]

$$[\tau_m(e^{j\omega})]^+ = C + \sum_{n=1}^{\infty} nu^+(n)\cos(\omega n), \tag{3.4}$$

$$[\tau_m(e^{j\omega})]^- = C + \sum_{n=1}^{\infty} nu^-(n)\cos(\omega n), \tag{3.5}$$

C is the average value that does not contribute to the shape of the spectrum. Cepstrally smooth spectra for poles and zeros can be obtained separately by considering only the first few cepstral coefficients in $u^+(n)$ and $u^-(n)$.

3.2. Modified magnitude group delay (MGDM) [10]

In spectral estimation, the white noise driving a system/associated with the system output or a signal or the truncation effect on the signal, introduces spectral ripple and significantly contributes to the variance of the spectral estimate. This spectral ripple manifests as zeros close to the unit circle in the Z-plane. The ripple effect/variance cannot be reduced by normal smoothing using a window *without any loss of frequency resolution*. The modified group delay [10] removes these zeros without disturbing the signal /system poles and hence reduces the variance preserving the frequency resolution.

The modification basically considers the signal to be characterized by an all-pole model. With this, the variance contribution corresponds to the numerator. Hence the variance effect can be removed by dividing the transfer function by the numerator estimate, without significantly disturbing the denominator. In the GD domain, this operation can be realized only by multiplications and hence there will not be any singularity problems due to divisions. This method of variance reduction preserves the frequency resolution (of a rectangular window) and is unlike that of periodogram averaging where the variance reduction is only at the cost of the frequency resolution due to the fact that the data windowing pulls not only the zeros close to the unit circle, but also the signal poles towards the origin.

If $x(n)$ is generated by an all-pole system, driven by a white noise input or has sinusoids with white noise and $X(e^{j\omega}) = N(e^{j\omega})/D(e^{j\omega})$ then $D(e^{j\omega})$ corresponds to the system and $N(e^{j\omega})$ to excitation or the observation noise. For this, it has been shown that [10]

$$\tau_m(e^{j\omega}) = \frac{K_N}{|N(e^{j\omega})|^2} - \frac{K_D}{|D(e^{j\omega})|^2}, \tag{3.6}$$

where K_N and K_D are constants (say). For the zeros close to unit circle, the first term in (3.6) will be of large amplitude due to very small values of $|N(\omega)|^2$ and this is not so with the second term as $|D(e^{j\omega})|^2$ is sufficiently large for the poles well within the unit circle. Hence, the first term will mask the second term in (3.6). The effect of these zeros can be reduced by multiplying $\tau_m(e^{j\omega})$ by $|N(e^{j\omega})|^2$. Hence, the modified MGD (MGDM) $\tau_{mo}(\omega)$ is

$$\tau_{mo}(e^{j\omega}) = \tau_m(e^{j\omega})|\hat{N}(e^{j\omega})|^2, \tag{3.7}$$

$$|\hat{N}(e^{j\omega})|^2 = |X_w(e^{j\omega})|^2/|\tilde{X}_w(e^{j\omega})|^2 \tag{3.8}$$

$|\hat{N}(e^{j\omega})|^2$ is the estimate of $|N(e^{j\omega})|^2$, $|\tilde{X}(e^{j\omega})|^2$ is smoothed spectrum of the signal obtained by the truncated cepstral coefficient sequence. The magnitude spectrum derived from the MGDM has to be scaled with reference to the original spectrum.

4. Channel/system identification based on group delay

As pointed out in the Section 2.2, the phase estimated is erroneous and the phase correction can be made both for the poles and zeros. For poles, the phase correction applied by parametric methods [8] may severely get affected when the additive noise is strong. Presently it has been proposed to apply in the group delay domain, the

phase correction and also derive an improved spectral magnitude estimate, due to its simplicity and less sensitivity to the additive noise.

4.1. NP method with phase correction by magnitude group delay (NP-MGD)

For a channel/system (with an i.i.d four-level PAM input $a(n)$ and with an additive output Gaussian white noise $w(n)$), the over-sampled output $y(n)$ by a factor P, is obtained over-sampling the input sequence $a(n)$ by inserting $(P-1)$ zeros between samples. Depending upon the scenario, the SCD, $S_y^{\alpha}(\omega)$ can be estimated by choosing any one of the appropriate method available in [3]. In the present study, the SCD is computed by smoothing the cyclic periodogram, $I_x(\omega,\alpha)$ using a spectral window $\xi(\omega)$ of length K (K determines the smoothing).

$$S_y^{\alpha}(\omega)=\frac{1}{K}\left[\sum_{k=-(K-1)/2}^{(K-1)/2}\xi(k)I_y(\omega-k,\alpha)\right],$$

where $I_y(\omega,\alpha)=1/N\{Y(\omega-\alpha/2)Y^*(\omega+\alpha/2)\}$, $Y(\omega)=\mathrm{FT}[y(n)]$.

The magnitude spectrum from the SCD can be computed for no noise case and for noise case using Eqs. (2.7) and (2.9), respectively. Further from this magnitude $|\hat{H}(\omega)|$, the MGD $\tau_m(\omega)$ can be computed using Eq. (3.1a) and Eq. (3.3), i.e.,

$$c_1(n)=\mathrm{IDFT}[\ln|\hat{H}(\omega)|],$$

$$c(n)=\begin{cases}c_1(n), & n=0,L/2,\\ 2c_1(n), & n=1,2,\ldots,L/2-1,\\ 0, & n=L/2+1,\ldots,L-1,\end{cases}$$

$$\tau_m(\omega)=\mathrm{Real}[\mathrm{DFT}\{nc(n)\}],$$

where L is the DFT length. Let $[\tau_m(\omega)]^+$ and $[\tau_m(\omega)]^-$ represent the positive and negative parts of $\tau_m(\omega)$, respectively.

The PGD, $\tau_p(\omega)$ can be computed from the phase derived from SCD (using Eq. (2.11)) by computing the all-pass sequence and using Eq. (3.2b). If the unwrapped phase is known $\tau_p(\omega)$ can be computed using Eqs. (3.1b) and (3.2a). Let $[\tau_p(\omega)]^+$ and $[\tau_p(\omega)]^-$, be the positive and negative parts of $\tau_m(\omega)$, respectively.

The errors in estimating the phase by nonparametric approach (Section 2), affect the location of both poles and zeros. For a physically realizable system, all the poles must be of minimum phase and the poles correspond to the positive portion of the MGD (Eq. (3.4)). The MGD will contain complete information about the poles. In the PGD, the correction for the poles [9] is made as

$$[\tau_p(\omega)]^+=[\tau_m(\omega)]^+ \quad \text{if } \tau_m(\omega)>0.$$

However, phase correction for zeros requires the information about their locations with respect to unit circle whether they are inside (minimum phase) or outside (maximum phase). The PGD provides this information, as the PGD is positive for a maximum phase zero and negative for a minimum phase zero. For a minimum phase zero, the phase correction is achieved by

$$[\tau_p(\omega)]^-=[\tau_m(\omega)]^- \quad \text{if } \tau_m(\omega)<0 \text{ and } \tau_p(\omega)<0.$$

For a maximum phase zero, the positive part of PGD (other than that for poles) is replaced with the corresponding negative part of MGD with change in sign. Let $\Gamma(\omega)=[\tau_p(\omega)]^+-[\tau_m(\omega)]^+$ then,

$$[\tau_p(\omega)]^+=-[\tau_m(\omega)]^- \quad \text{if } \tau_m(\omega)<0 \text{ and } \Gamma(\omega)>0,$$

For the corrected PGD, $\tau_{\mathrm{pc}}(\omega)$, $q_1(k)=\mathrm{IDFT}[\tau_{\mathrm{pc}}(\omega)]$,

$$\lambda(k)=\begin{cases}0, & k=0,L/2,\\ 2q_1(k)/k, & k=1,2,\ldots,L/2-1,\\ 0, & k=L/2+1,\ldots,L-1,\end{cases}$$

where $\lambda(k)$ are the Cepstral coefficients derived from the corrected PGD. L is the DFT length. These corrections may result in some discontinuity in the corrected PGD $\tau_{\mathrm{pc}}(\omega)$ and can be reduced by considering only first few Fourier coefficients $M(M\ll L)$ of the PGD and the smoothed corrected PGD $\tilde{\tau}_{\mathrm{pc}}(e^{j\omega})$ is

$$\tilde{\tau}_{\mathrm{pc}}(e^{j\omega})=\sum_{n=1}^{M}k\lambda(k)\cos(\omega k). \tag{4.1}$$

Then, the system phase is

$$\hat{\phi}(e^{j\omega}) = -\sum_{n=1}^{M} \lambda(n)\sin(\omega n). \quad (4.2)$$

This corrected phase can be used for the channel identification along with the estimated magnitude. This method of phase correction is applicable even in the presence of the associated noise. However, additional frequency smoothing for SCD is required to reduce the variance due to noise, and this may deteriorate the frequency resolution and hence the performance. Here phase correction is both for poles and zeros, however, the approach in [9] is only for poles.

The system spectral magnitude $|\hat{H}(e^{j\omega})|$ obtained is associated with the estimated phase $\hat{\phi}(e^{j\omega})$ to get the frequency response of the system $\hat{H}(e^{j\omega})$

$$\hat{H}(e^{j\omega}) = |\hat{H}(e^{j\omega})|e^{j\hat{\phi}(\omega)} \quad \text{and}$$

$$\hat{h}(n) = \text{IDFT}[\hat{H}(e^{j\omega})].$$

The number of cepstral coefficients used should be sufficient to get only the gross feature of the spectrum as the cepstral coefficients are the Fourier coefficients of the log spectrum. As their number increases, the FT of the truncated cepstrum will be closer to the original spectrum (i.e., which includes all the cepstral coefficients). The normalized error energy $E_n(m)$ considering only the first q cepstral coefficients $c(k)$ is

$$E_n(m) = \sum_{k=m+1}^{\infty} c^2(k) \Big/ \sum_{k=1}^{\infty} c^2(k).$$

Initially $E_n(m)$ will decrease sharply as m is increased but after a certain value of $m = M$, *the knee point*, the decrease is not appreciable. This implies that M cepstral coefficients provide a good representation of the spectrum.

4.2. Channel/system identification based on non-parametric method and modified group delay (NP-MGDM)

In the presence of noise, to reduce the variance of the phase estimate, the procedure described in Section 4.1 demands high level of spectral smoothing. This severely affects the frequency resolution of the phase estimate and hence at high noise levels, the MGDM that reduces the variance without reducing the frequency resolution, is expected to be very appropriate.

In the presence of strong noise, the MGD $\tau_{mi}(\omega)$ obtained from the SCD (with limited frequency smoothing) using Eqs. (2.7), (3.1a) and (3.3) will have a spectral envelope with fluctuations superimposed on it. These fluctuations can be significantly reduced by estimating the MGDM $\tau_{Do}(\omega)$ using Eq. (3.7) without sacrificing frequency resolution. This MGDM $\tau_{Do}(\omega)$ corresponds to the denominator. As described in Section 4.1, the phase correction for the denominator of the system (poles) can be done in GD domain using MGDM $\tau_{Do}(\omega)$. However, in removing the fluctuations present in MGD, the part of the MGD corresponding to the numerator of the system (zeros), is also removed. Unless the system/signal zeros are damped (away from the unit circle), they are also removed by the MGDM, as it cannot distinguish between wanted (of the system) and unwanted zeros. So to correct the phase for the system numerator (zeros), the MGDM $\tau_{No}(\omega)$ has been derived by considering the original MGD with its sign changed, i.e. $-\tau_{mi}(\omega)$.

From the MGDMs of the denominator $\tau_{Do}(\omega)$ and of the numerator $\tau_{No}(\omega)$, the corresponding system magnitude estimates $|\hat{H}_N(\omega)|$ and $|\hat{H}_D(\omega)|$ (which are scaled with respect to original magnitude derived from Eq. (2.7)) which are having a reduced variance without any loss in frequency resolution can be obtained (Eqs. (3.3) and (3.1a)). From the improved magnitude spectra $|\hat{H}_N(\omega)|$ and $|\hat{H}_D(\omega)|$, the new system magnitude $|\hat{H}(\omega)| = |\hat{H}_N(\omega)|/|\hat{H}_D(\omega)|$, and the new MGDs $\tau_{DS}(\omega)$, $\tau_{NS}(\omega)$ and $\tau_{mo}(\omega) = \tau_{DS}(\omega) - \tau_{NS}(\omega)$ are derived.

The phase correction method is the same, except that it is done using an improved MGD $\tau_{mo}(\omega)$ of lesser variance but without any loss of frequency resolution derived as above.

Thus in this method, both improved system phase and magnitude estimates obtained using MGDM will be used in identifying the channel/system. The initial spectral magnitude estimate derived using Eq. (2.7) has to be used in computing the MGDM, as its performance is better in the presence of noise.

For the MGDM, in estimating $|\hat{N}(e^{j\omega})|^2$ using Eq. (3.8b), as only the smoothed spectral estimate $|\tilde{X}(e^{j\omega})|^2$ is required, the number of cepstral coefficients to be used should be much less than that corresponding to the knee point of normalized error energy. To have a low variance, this number should be kept small. Based on this, for the smoothed spectrum the choice of number of cepstral coefficients is made and this number is not very critical. It can be in the range of 5–10 without much difference in performance [10]. The MGDM is valid even for the case of high signal to noise ratio, when $|\tilde{X}(e^{j\omega})|^2$ is computed by using a large number of initial cepstral coefficients.

It was stated that the NP method which is only based on cyclostationarity or SCD cannot identify *completely* any arbitrary channel. Further, as it has been reported [8] the channel that has zeros located at uniform spacing along a circle cannot be identified by a SCD-based method, since the proposed methods belong to this class, they also have this limitation But for the case where the poles are uniformly spaced on a circle it has been shown in [8] that the application of phase correction for the pole part has improved the performance. In view of this, it is expected that the proposed approach, which applies phase correction both for poles and zeros, will significantly improve the performance. It should be noted that the group delay methods are applicable only for those signals whose poles and zeros are not on the unit circle.

5. Simulation results

The simulation study has been carried out for three channels, both with and without noise cases. For the no noise case, the data length, oversampling rate P and the number of trials Q (considered for averaging) used are 4096, 4 and 50, respectively.

For the case with noise, the corresponding parameters used are 8192, 3 and 20, respectively. In both cases, the input i.i.d. sequence used is a four-level PAM signal with symbols $\{-3,-1,1,3\}$. For the noise case, for the NP-MGDM method, the system spectral magnitude estimate is derived from the MGDM for the poles and zeros. The performance indices are:

$$\hat{h}(n) = (1/Q)\sum_{i=1}^{Q} \hat{h}_i(n),$$

$$\sigma^2(n) = (1/Q)\sum_{i=1}^{Q} [h(n) - \hat{h}_i(n)]^2,$$

$\%\text{NMSE} = 100[\sigma^2(n)/\sum_{n=0}^{\infty}|h(n)|^2]$, $\hat{h}(n)$ is the mean, $\hat{h}_i(n)$ the estimate of $h(n)$, $\sigma(n)$ the RMSE, and %NMSE the percentage normalized mean square error.

Channel-I:

$$H_1[z] = \frac{1 + 1.50000z^{-1}}{1 - 1.64453z^{-1} + 0.740818z^{-2}}.$$

The channel has poles at $0.82227 \pm j0.25436$ and a zero at -1.500. The value of M used is 20. For the SCD, a frequency domain boxcar smoothing is done over 51 points, for NP and NP-MGD. The performance of system identification is shown in Fig. 2. The sample mean of the impulse response estimate by the NP-MGD has a better match with the true one than that by the existing NP method. The RMSE plot indicates that variance of the NP-MGD is significantly less than that for NP (Fig. 2). The %NMSE obtained with NP and NP-MGD, respectively, are 8.90 and 1.26 indicating an improvement of 85%.

The performance of the proposed methods for the same example is considered for different signal to noise ratios, SNR = 20, 10 and 0 dB. For the SCD, a frequency domain boxcar smoothing is done for NP and NP-MGDM for noise case over 201 points. For SNR = 20 dB, the performance results are shown in Fig. 3 and the %NMSE by NP method is 14.93 and that by NP-MGDM is 5.27, indicating an improvement of 65% over that of NP. For the proposed NP-MGDM method, in computing the smoothed spectral envelope the first 13 cepstral coefficients and the value of $M = 20$ are considered.

For SNR = 10 dB (Fig. 4) and 0 dB (Fig. 5), the results show that the performance of NP method, specifically at SNR = 0 dB is very poor both in terms of sample mean and variance. But the proposed NP-MGDM method due to phase correction both for poles and zeros, and improved

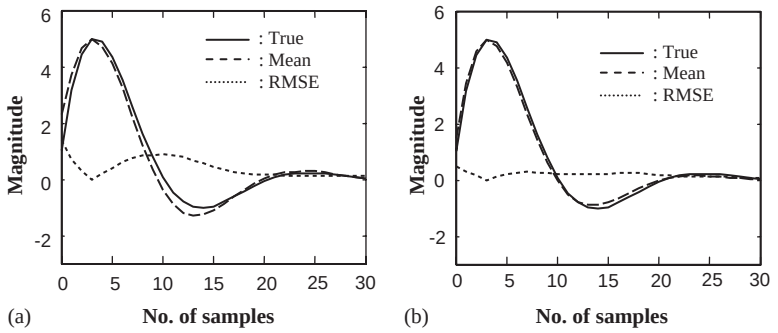

Fig. 2. System blind identification without noise case for channel-I: (a) by NP method; (b) by NP-MGD method.

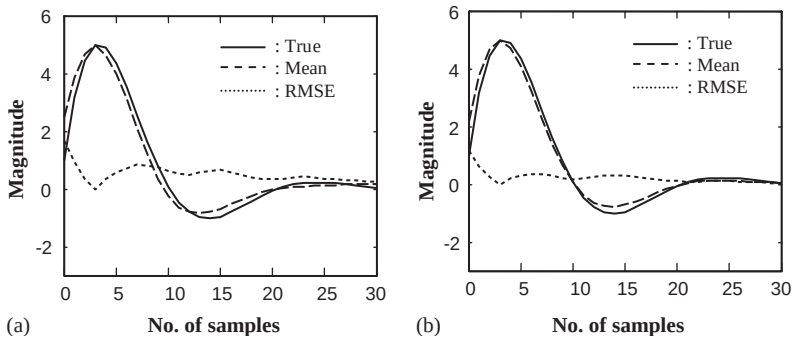

Fig. 3. System blind identification with noise at SNR = 20 dB of channel-I: (a) by NP method; (b) by NP-MGDM method.

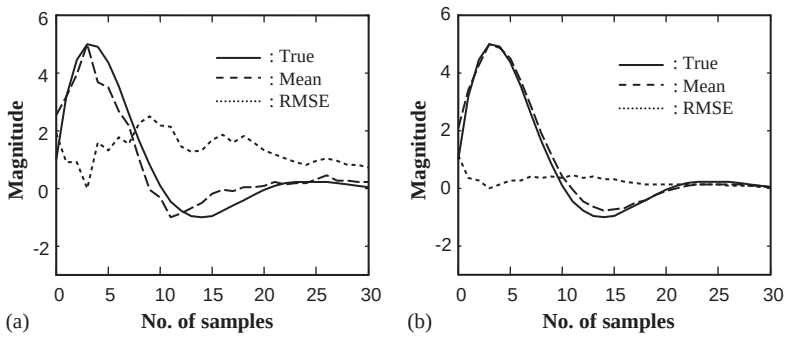

Fig. 4. System blind identification with noise at SNR = 10 dB of channel-I: (a) by NP method; (b) by NP-MGDM method.

system magnitude estimate, provides a significantly better performance over that of NP.

Channel-II :

$$H[z] = \frac{1 - 0.4z^{-1} + 0.96z^{-2}}{1 - 0.9z^{-1} + 0.81z^{-2}}.$$

This channel has poles at $-0.4500 \pm j0.7794$ and zeros at 0.8000 and -1.200. For the NP-MGD, the value of M chosen is 20 and the frequency smoothing for the SCD is over 51 points. The %NMSE with NP and NP-MGD methods are 24.35 and 1.00, respectively, i.e., an improvement

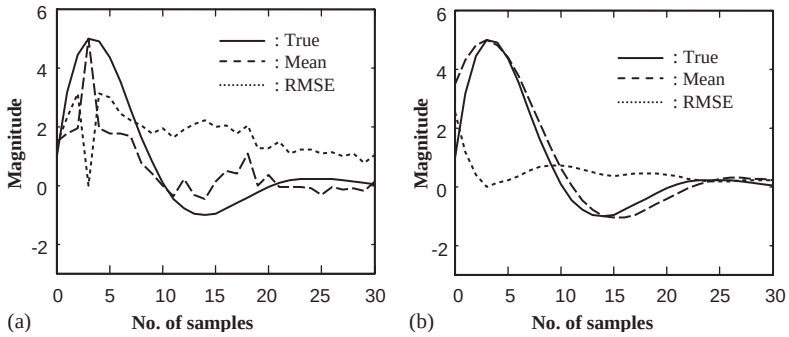

Fig. 5. System blind identification with noise at SNR = 0 dB of channel-I: (a) by NP method; (b) by NP-MGDM method.

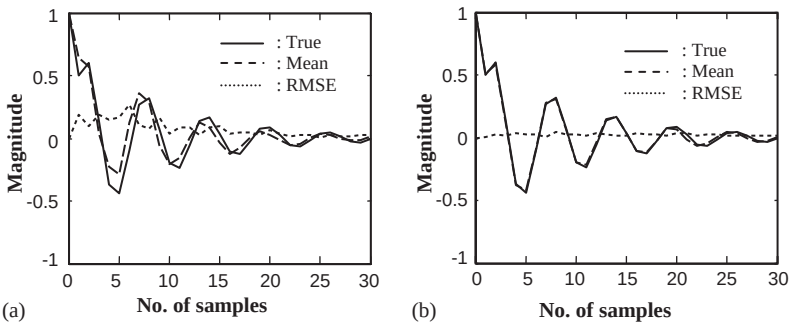

Fig. 6. System blind identification without noise case of channel-II: (a) by NP method; (b) by NP-MGD method.

of 96%. The NP-MGD non-parametric method outperforms the existing method (Fig. 6).

In the presence of noise at SNR = 12 dB, the frequency smoothing of SCD, for NP and MGDM is done over 201 points. Further, MGDM is obtained by computing envelope with first 20 coefficients and the value of M used is 20. The %NMSE calculated by NP and NP-MGDM are 47.43 and 12.37, respectively (an improvement of 74%) and the results are shown in Fig. 7. The mean and reduction in variance obtained by the proposed NP-MGDM method are better compared to existing method.

Channel-III :

$$H[z] = \frac{1 + 0.6000z^{-1} - 0.3937z^{-3}}{1 - 0.6561z^{-4}}.$$

This channel has four poles at $\pm 0.6364 \pm$ j0.6364 and three zeros at $-0.5890 \pm$ j0.5780 and 0.5781. The frequency smoothing is done over 51 points and $M = 20$ are used. The %NMSE for NP and NP-MGD are 14.81 and 1.05, hence an improvement of 93% over the NP (Fig. 8).

The channel-III has poles uniformly placed on a circle and hence use of only non-parametric method results in a serious phase distortion (Fig. 9) as phase information is not available at these sampling instants. At SNR = 5 dB, the results of NP and NP-MGDM methods are shown in Fig. 9. The frequency smoothing for SCD, for MGDM and NP is over 201 points. In estimating the MGDM, the smoothed spectral envelope is obtained by considering the first 8-cepstral coefficients and the value of M used is 20. The results for the combination of NP and parametric method (used for correcting the phase for poles) (NPP)[8] are shown in Fig. 9b.

For NPP also, the frequency smoothing required for NPP is over 201 points. The %NMSE for NP, NPP and NP-MGDM are 61.66, 38.77

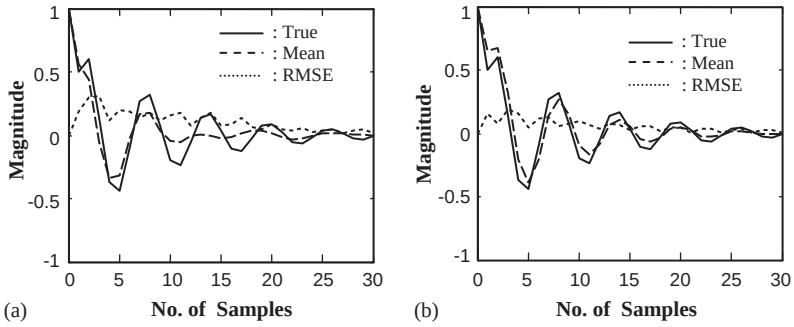

Fig. 7. System blind identification with noise at SNR = 12 dB of channel-II: (a) by NP method; (b) by NP-MGDM method.

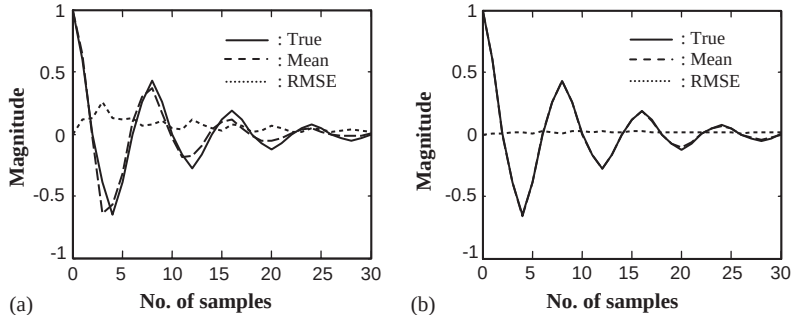

Fig. 8. System blind identification without noise case of channel-III: (a) by NP method; (b) by NP-MGD method.

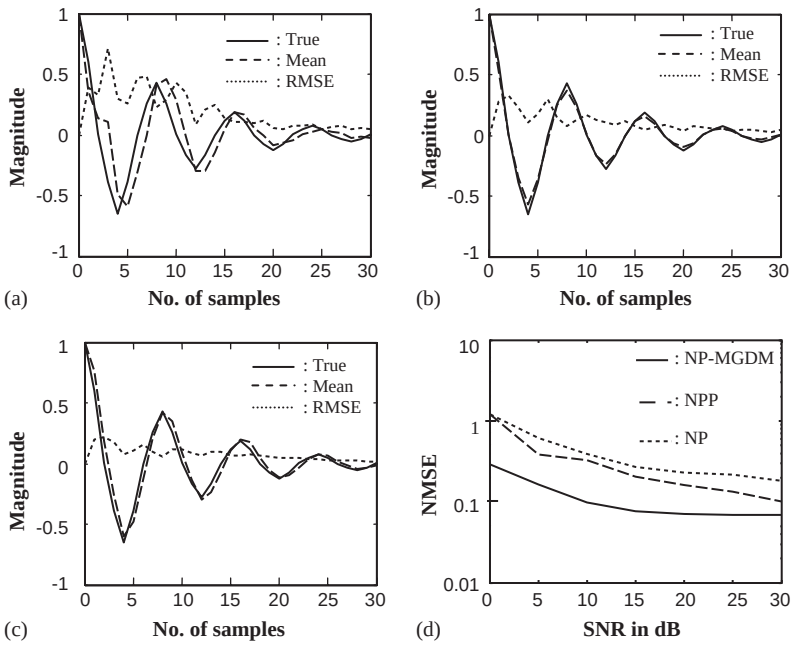

Fig. 9. System blind identification with noise at SNR = 5 dB of channel-III: (a) by NP; (b) by NPP; (c) NP-MGDM; and (d) NMSE vs. SNR for various methods.

and 16.64, respectively and this implies an improvement of NP-MGDM about 73% over NP and 57% over NPP.

It should be noted that because of the missing information, the performance of NP is poor. But for the NPP as it applies phase correction to the pole part, its performance is improved. The proposed method NP-MGDM as it applies phase correction *both for poles and zeros*, its improvement over that of NPP and NP is very significant.

For Channel-III, Fig. 9d shows the %NMSE of different methods with SNR variation. As the SNR increases from 0 dB, the performance of NP and the NPP, improves as the NMSE decreases but their performance is significantly poorer than that of NP-MGDM and particularly at SNR below 10 dB.

6. Conclusions

A new approach for system/channel blind identification based on second order cyclostationary statistics and the group delay, was proposed. In this, in both the methods proposed, to the basic phase estimate, correction *for both poles and zeros* is applied. The basic system phase and magnitude estimates are derived from the spectral correlation density of the system/channel output by the non-parametric method. In the first method, the phase correction is based on magnitude group delay and in the second, on the modified magnitude group delay. In addition to the phase correction, an improved spectral magnitude derived modified magnitude group delay was used for the second method. For the first method the %NMSE is 85% less than the existing nonparametric method. The second method is effective even at low SNR as the modified group delay reduces the effect of noise without requiring a high degree of frequency domain smoothing. Its %NMSE is 74% less than the non-parametric method and 57% less than the combined parametric and non-parametric methods.

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