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COST ANALYSIS OF A TWO-UNIT REPAIRABLE SYSTEM SUBJECT TO ON-LINE PREVENTIVE MAINTENANCE AND/OR REPAIR

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Abstract—This paper deals with the cost analysis of a two-unit repairable system subject to on-line preventive maintenance (on-line PM) and/or repair. The policy adopted here is that the on-line PM work of the operating unit is undertaken first on its completion, the repair work of the failed unit, if any, is subsequently carried out. All the random variables that arise in the analysis are assumed to be independently and arbitrarily distributed. An expression for the expected total cost incurred by the system in a specified time interval is obtained by considering the expected busy period of the server spent on various actions. The analysis is carried out using the regeneration point technique.

1. INTRODUCTION

It is well known that preventive maintenance improves the reliability of a system. In most of the literature reported so far on the preventive maintenance aspect of a system, it is assumed that the system is shut down while the maintenance work is undertaken. In many situations, shut-down or off-line maintenence is uneconomical to the organisations concerned, in the form of wastage of raw materials or a wastage of time in attaining the peak load, which may result in reduced output. In such cases, carrying out on-line preventive maintenance [1] (the preventive maintenance that is performed whilst the system or plant is in operation) would be advantageous as this would increase the system availability.

Gopalan and Murulidhar [2] have recently discussed the analysis of a one-unit system subject to on-line PM and/or repair, and have obtained explicit expressions for the system characteristics required when carrying out the cost analysis of the system. Several strategies of maintenance and repair have been considered by the authors in their paper.

The present paper deals with the analysis of a one-server two-unit cold standby system and the policy adopted is that the operating unit is subject to on-line PM followed by the repair work of the failed unit (if any).

A system consisting of two identical units is considered. Initially, one unit starts operating and the other is kept as a cold standby. The system is provided with a single-server facility which carries out on-line inspection, on-line PM, inspection (associated with repair) and the repair of a failed unit.

If the system is operating when the server arrives, he first takes up the operating unit for maintenance work. After this job is completed, he takes up repair work of the failed unit (if any), upon completion of which, he departs and revisits according to his arrival schedule. It is assumed that, upon the completion of the on-line PM action/repair work, a unit attains its original condition.

Explicit expressions have been obtained for the expected busy period of the server spent on on-line inspection, on-line PM action of type i ($i = 1$ to r), inspection (associated with repair) and repair work of type j ($j - 1$ to s) in the interval $[0, t]$. Integral equations have been written by identifying the system at suitable regeneration epochs [3]. Finally, an expression for the expected total cost to be incurred on the system in $[0, t]$ has been obtained.

2. ASSUMPTIONS

(1) Both units in the system are identical in all respects with identical failure densities.
(2) A unit will not fail while it is undergoing on-line inspection or on-line preventitive maintenance.
(3) The system can undergo r different types of PM actions and s different types of failures.

(4) Every on-line inspection/inspection associated with repair, leads to on-line preventive maintenance action/repair work with a respective probability attached to it.
(5) The server takes up PM action of type i with a probability p_i,

$$\sum_{i=1}^{r} p_i = 1.$$

(6) The server takes up repair work of type j with a probability q_j,

$$\sum_{j=1}^{s} q_j = 1.$$

(7) Switchover from the arrival of the server to on-line inspection/inspection associated with repair, and to appropriate follow-up actions is assumed to be instantaneous.
(8) The server resorts to only one type of on-line PM action/repair work during a visit and on its completion, he departs and revisits according to his arrival schedule.
(9) The lifetime of the system, the arrival time of the server, the time spent on on-line inspection, on-line PM, inspection associated with repair and repair work are all independently and arbitrarily distributed.

3. NOTATION

Pr Probability

pdf, sf Probability density function, survivor function

$g(t)\,dt$ $Pr\cdot$ that the server arrives during the interval $[t, t+dt]$

$f(t)\,dt$ $Pr\cdot$ that a unit fails during the interval $[t, t+dt]$

$k_i(t)\,dt$ $Pr\cdot$ that the server completes on-line PM action of type i ($i = 1$ to r) during the interval $[t, t+dt]$

$l(t)\,dt$ $Pr\cdot$ that the server completes inspection (associated with repair) during the interval $[t, t+dt]$

$m_j(t)\,dt$ $Pr\cdot$ that the server completes repair work of a type j ($j = 1$ to s) in the interval $[t, t+dt]$

p_i $Pr\cdot$ with which server resorts to on-line PM action of a type i ($i = 1$ to r),

$$\sum_{i=1}^{r} p_i = 1$$

q_j $Pr\cdot$ with which server resorts to repair work of a type j ($j = 1$ to s),

$$\sum_{j=1}^{s} q_j = 1$$

$\bar{F}(t), \bar{G}(t), \bar{H}(t), \bar{K}_i(t), \bar{L}(t), \bar{M}_j(t)$ sfs corresponding, respectively, to the pdfs $f(t), g(t), h(t), k_i(t), l(t), m_j(t)$

$*$ Convolution,

$$f(t) * g(t) = \int_0^t f(u)g(t-u)\,du$$

$\bar{\psi}(s)$ LT of the function $\psi(t)$

$$= \int_0^\infty e^{-st}\psi(t)\,dt$$

E Initial state of the system

n State of the system

$X(t)$ State variable characterising the state of the system at an instant t

$AV_n^E(t)$ $Pr[X(t) = n/E]$

OI, M(i), IR, R(j) States of the system corresponding to on-line inspection, on-line PM work of type i ($i = 1$ to r), inspection (associated with repair) and repair work of type j ($j = 1$ to s), respectively

$\mu_{OI}^E(t)$ Expected busy period of the server due to on-line inspection carried out in $[0, t]$

$$= \int_0^t AV_{OI}^E(u)\,du$$

$\mu_{M(i)}^E(t)$ Expected busy period of the server due to on-line PM action of types i ($i = 1$ to r) carried out in $[0, t]$

$$= \int_0^t AV_{M(i)}^E(u)\,du$$

$\mu_{IR}^E(t)$ Expected busy period of the server due to an inspection (associated with repair) carried out in $[0, t]$

$$= \int_0^t AV_{IR}^E(u)\,du$$

$\mu_{R(j)}^E(t)$ Expected busy period of the server due to repair work of type j ($j = 1$ to s) carried out in $[0, t]$

$$= \int_0^t AV_{R(j)}^E(u)\,du$$

C_{OI} Cost of on-line inspection per unit time

$C_{M(i)}$ Cost of on-line PM action of type i ($i = 1$ to r) per unit time

C_{IR} Cost of inspection (associated with repair) per unit time

$C_{R(j)}$ Cost of repair work of type j ($j = 1$ to s) per unit time

$a_1(t)$ $\bar{F}(t)g(t)$

$a_2(t)$ $g(t)[f(t) * \bar{F}(t)]$

$a_3(t)$ $g(t)[f(t) * F(t)]$

$a_4^j(t)$ $q_j\bar{F}(t)[l(t) * m_j(t)]$

$a_5(t)$ $F(t)l(t)$

$a_6^j(t)$ $q_j \int_0^t l(u)m_j(t-u)\,du \int_0^{t-u} f(x+u)\,dx$

$a_7^j(t)$ $q_j\bar{F}(t)[l(t) * \bar{M}_j(t)]$

$a_8^j(t)$ $q_j \int_0^t l(u)\bar{M}_j(t-u)\,du \int_0^{t-u} f(x+u)\,dx$

$a_9(x, t)$ $g(t)\bar{F}(x+t)$

$a_{10}(x, t)$ $g(t) \int_0^t f(x+x_1)\bar{F}(t-x_1)\,dx_1$

$a_{11}(x, t)$ $g(t) \int_0^t f(x+x_1)F(t-x_1)\,dx_1$

$a_{12}(t)$ $l(t)\bar{F}(t)$

$b_1^j(t)$ $q_j \int_0^t l(u)m_j(t-u)\,du$

$b_2^j(t)$ $q_j[l(t)F(t)]$

$\tilde{a}_1(p, s)$ Double LT of

$$\int_0^t a_9(x, u)AV_{OI}^1(t-u)\,du$$

$$= \int_0^\infty \int_0^\infty e^{-px-st}\,dx\,dt \times \int_0^t g(u)\bar{F}(x+u)AV_{OI}^1 \times (t-u)\,du$$

$\tilde{\tilde{a}}_2(p,s)$ Double LT of

$$\int_0^t a_{10}(x,u)AV^4_{OI}(t-u)\,du$$

$\tilde{\tilde{a}}_3(p,s)$ Double LT of

$$\delta\int_0^t a_{11}(x,u)AV^9_{OI}(t-u)\,du$$

$\tilde{\tilde{A}}_i(p,s)$ $\int_0^\infty\int_0^\infty e^{-px-su}a_k(x,u)\,du\,dx$

$(i,k)=(1,9),\ (2,10),\ (3,11)$

$$C_i(s)=\sum_{j=1}^{s}\int_0^\infty e^{-su}b_1^j(u)L^{-1}[\tilde{\tilde{A}}_i(p,s),p]\,du,$$

$i=1,2,3$

$$C_4(s)=\sum_{j=1}^{s}\tilde{a}_6^j(s)+\sum_{j=1}^{s}\tilde{b}_2^j(s)\tilde{m}_j(s)$$

$$C_5(s)=[\tilde{a}_1(s)+\tilde{a}_2(s)]\tilde{\bar{H}}(s)$$

$$C_6(s)=\sum_{i=1}^{r}\tilde{h}(s)\tilde{k}_i(s)\tilde{a}_2(s)+\sum_{j=1}^{s}q_j\tilde{l}(s)\tilde{a}_3(s)\tilde{m}_j(s)$$

$$C_7(s)=[C_1(s)+C_2(s)]\tilde{\bar{H}}(s)$$

$$C_8(s)=C_1(s)\sum_{i=1}^{r}p_i\tilde{h}(s)\tilde{\bar{K}}_i(s)$$

$$C_9(s)=[\tilde{a}_1(s)+\tilde{a}_2(s)]p_i\tilde{h}(s)\tilde{\bar{K}}_i(s)$$

$$C_{10}(s)=[C_1(s)+C_2(s)]p_i\tilde{h}(s)\tilde{\bar{K}}'_1(s)$$

$$C_{11}(s)=\tilde{a}_3(s)\tilde{\bar{L}}(s)$$

$$C_{12}(s)=\tilde{\bar{L}}(s)[1+C_3(s)]$$

$$C_{13}(s)=q_j\tilde{l}(s)\tilde{a}_3(s)\tilde{\bar{M}}_j(s)$$

$$C_{14}(s)=\tilde{b}_2^j(s)\tilde{\bar{M}}_j(s)+q_j[\tilde{a}_{12}(s)\tilde{\bar{M}}_j(s)]+C_3(s)q_j\tilde{\bar{M}}_j(s)\tilde{l}(s)$$

$$D_1(s)=1-\sum_{i=1}^{r}p_i\tilde{h}(s)\tilde{k}_i(s)\tilde{a}_1(s)$$

$$D_2(s)=1-C_4(s)-\sum_{i=1}^{r}p_i\tilde{h}(s)\tilde{k}_i(s)C_2(s)-\sum_{j=1}^{s}q_j\tilde{l}(s)\tilde{m}_j(s)C_3(s)$$

δ_{ij} Kronecker's delta

4. ANALYSIS

4.1. *Busy period analysis*

In this section, explicit expressions are obtained for the expected busy period of the server in $[0, t]$ due to on-line inspection, on-line PM action of type i ($i = 1$ to r), inspection (associated with repair), and repair work of type j ($j = 1$ to s), respectively.

State space of the system. The system can be found in any of the following states at an instant t (Table 1). The one-step transitions that the system can make between the states 0 and $10(j)$ are shown in Fig. 1.

Expected busy period of the server due to on-line inspection carried out in [0, t]. We have

$$AV^0_{OI}(t)=AV^0_1(t)+AV^0_4(t)$$
$$=[a_1(t)*AV^1_{OI}(t)]+[a_2(t)*AV^4_{OI}(t)]+[a_3(t)*AV^9_{OI}(t)]$$

$$AV^1_{OI}(t)=\bar{H}(t)+\sum_{i=1}^{r}p_i[h(t)*AV^{2(i)}_{OI}(t)]$$

Table 1.

Description of the state		
$X(t)$	Unit 1	Unit 2
0	OPG	SB
1	OI	SB
$2(i)$	OPM	SB
3	WR	OPG
4	WR	OI
$5(i)$	WR	PM
6	IR	OPG
$7(j)$	RW	OPG
8	WR	WR
9	IR	WR
$10(j)$	RW	WR

OPG: operating, OI: on-line inspection, SB: standby, OPM: on-line preventive maintenance, IR: inspection due to repair, RW: repair work, WR: waits for repair.

$$AV^{2(i)}_{OI}(t)=k_i(t)*AV^0_{OI}(t)$$

$$AV^4_{OI}(t)=\bar{H}(t)+\sum_{i=1}^{r}p_i[h(t)*AV^{5(i)}_{OI}(t)]$$

$$AV^{5(i)}_{OI}(t)=k_i(t)*AV^6_{OI}(t)$$

$$AV^6_{OI}(t)=\sum_{j=1}^{s}\int_0^t\phi^0_{OI}(u,t-u)b_1^j(u)\,du+\sum_{j=1}^{s}[a_6^j(t)*AV^6_{OI}(t)]+\sum_{j=1}^{s}[b_2^j(t)*AV^{10(j)}_{OI}(t)],$$

where

$$\phi^0_{OI}(x,t)=[a_9(x,t)*AV^1_{OI}(t)]+[a_{10}(x,t)*AV^4_{OI}(t)]+[a_{11}(x,t)*AV^9_{OI}(t)]$$

$$AV^9_{OI}(t)=\sum_{j=1}^{s}q_j[l(t)*AV^{10(j)}_{OI}(t)]$$

$$AV^{10(j)}_{OI}(t)=m_j(t)*AV^6_{OI}(t).$$

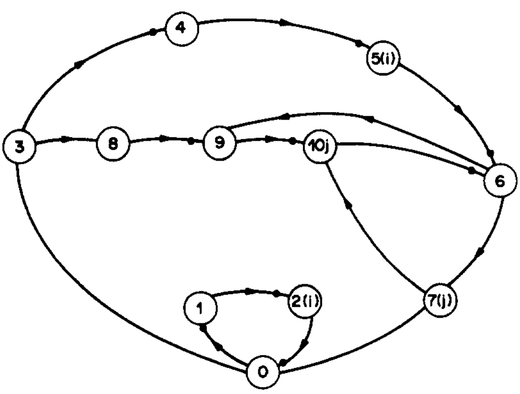

Fig. 1. Flow diagram showing the one-step transitions between the states 0 and $10(j)$.

Upon taking the LT of the above expressions and simplifying, we obtain,

$$\tilde{A}V^0_{OI}(s) = [C_5(s) + \tilde{A}V^6_{OI}(s)C_6(s)]/D_1$$

and

$$\tilde{A}V^6_{OI}(s) = [C_7(s) + C_8(s)\tilde{A}V^0_{OI}(s)]/D_1,$$

which, upon solving, yields

$$\tilde{A}V^0_{OI}(s) = [C_5(s)D_2 + C_7(s)C_6(s)]/[D_1D_2 - C_6(s)C_8(s)]$$

and

$$\tilde{A}V^6_{OI}(s) = [C_5(s)C_8(s) + C_7(s)D_1]/[D_1D_2 - C_6(s)C_8(s)].$$

After suitable substitution and inversion, $AV^0_{OI}(t)$ can be obtained. Now,

$$\mu^0_{OI}(t) = \int_0^t AV^0_{OI}(u)\,du.$$

The LT of $\cdot\mu^0_{OI}(t)$ is

$$\tilde{\mu}^0_{OI}(s) = \tilde{A}V^0_{OI}(s)/s,$$

which, upon inversion, will give $\mu^0_{OI}(t)$.

Expected busy period of the server due to on-line PM work of type i (i = 1 *to* r) *carried out in* [0, t]. We have

$$AV^0_{M(i)}(t) = [a_1(t) * AV^1_{M(i)}(t)] + [a_2(t) * AV^4_{M(i)}(t)] + [a_3(t) * AV^9_{M(i)}(t)]$$

$$AV^1_{M(i)}(t) = \sum_{i'=1}^{r} p_{i'}[h(t) * AV^{2(i')}_{M(i)}(t)]$$

$$AV^{2(i')}_{M(i)}(t) = \delta_{ii'}\bar{K}_i(t) + k_{i'}(t) * AV^0_{M(i)}(t)$$

$$AV^4_{M(i)}(t) = \sum_{i'=1}^{r} p_{i'}[h(t) * AV^{5(i')}_{M(i)}(t)]$$

$$AV^{5(i')}_{M(i)}(t) = \delta_{ii'}\bar{K}_i(t) + k_{i'}(t) * AV^6_{M(i)}(t)$$

$$AV^6_{M(i)}(t) = \sum_{j=1}^{s}\int_0^t b^j_1(u)\phi^0_{M(i)}(u, t-u)\,du + \sum_{j=1}^{s}[a^j_6(t) * AV^6_{M(i)}(t)] + \sum_{j=1}^{s}[b^j_2(t) * AV^{10(j)}_{M(i)}(t)],$$

where

$$\phi^0_{M(i)}(x, t) = [a_9(x, t) * AV^1_{M(i)}(t)] + [a_{10}(x, t) * AV^4_{M(i)}(t)] + [a_{11}(x, t) * AV^9_{M(i)}(t)]$$

$$AV^9_{M(i)}(t) = \sum_{j=1}^{s} q_j[l(t) * AV^{10(j)}_{M(i)}(t)]$$

$$AV^{10(j)}_{M(i)}(t) = m_j(t) * AV^6_{M(i)}(t).$$

Upon taking the Laplace transforms of the above expressions and simplifying, we obtain,

$$\tilde{A}V^0_{M(i)}(s) = [C_9(s) + C_6(s)\tilde{A}V^6_{M(i)}(s)]/D_1$$

and

$$\tilde{A}V^6_{M(i)}(s) = [C_{10}(s) + C_8(s)\tilde{A}V^0_{M(i)}(s)]/D_2.$$

This system of simultaneous equations gives,

$$\tilde{A}V^0_{M(i)}(s) = [C_9(s)D_2 + C_{10}(s)C_6(s)]/[D_1D_2 - C_6(s)C_8(s)]$$

and

$$\tilde{A}V^6_{M(i)}(s) = [C_9(s)C_8(s) + C_{10}(s)D_1]/[D_1D_2 - C_6(s)C_8(s)],$$

which, upon inversion, will give $AV^0_{M(i)}(t)$,

$$\mu^0_{M(i)}(t) = \int_0^t AV^0_{M(i)}(u)\,du.$$

The LT of $\mu^0_{M(i)}(t)$ is

$$\tilde{\mu}^0_{M(i)}(s) = \tilde{A}V^0_{M(i)}(s)/s.$$

Upon taking the inverse LT of this expression, $\mu^0_{M(i)}(t)$ can be obtained.

Expected busy period of the server due to inspection (associated with repair) carried out in [0, t]. We have

$$AV^0_{IR}(t) = AV^0_6(t) + AV^0_9(t)$$

$$AV^0_{IR}(t) = [a_1(t) * AV^1_{IR}(t)] + [a_2(t) * AV^4_{IR}(t)] + [a_3(t) * AV^9_{IR}(t)]$$

$$AV^1_{IR}(t) = \sum_{i=1}^{r} p_i[h(t) * AV^{2(i)}_{IR}(t)]$$

$$AV^{2(i)}_{IR}(t) = k_i(t) * AV^0_{IR}(t)$$

$$AV^4_{IR}(t) = \sum_{i=1}^{r} p_i[h(t) * AV^{5(i)}_{IR}(t)]$$

$$AV^{5(i)}_{IR}(t) = k_i(t) * AV^6_{IR}(t)$$

$$AV^6_{IR}(t) = \bar{L}(t) + \sum_{j=1}^{s}\int_0^t b^j_1(u)\phi^0_{IR}(u, t-u)\,du + \sum_{j=1}^{s}[a^j_6(t) * AV^6_{IR}(t)] + \sum_{j=1}^{s}[b^j_2(t) * AV^{10(j)}_{IR}(t)],$$

where

$$\phi^0_{IR}(x, t) = [a_9(x, t) * AV^1_{IR}(t)] + [a_{10}(x, t) * AV^4_{IR}(t)] + [a_{11}(x, t) * AV^9_{IR}(t)]$$

$$AV^9_{IR}(t) = \bar{L}(t) + \sum_{j=1}^{s} q_j[l(t) * AV^{10(j)}_{IR}(t)]$$

$$AV^{10(j)}_{IR}(t) = m_j(t) * AV^6_{IR}(t).$$

Upon taking the LT of these expressions and simplifying, we obtain

$$\tilde{A}V^0_{IR}(s) = [C_{11}(s) + C_6(s)\tilde{A}V^6_{IR}(s)]/D_1$$

$$\tilde{A}V^6_{IR}(s) = [C_{12}(s) + C_8(s)\tilde{A}V^0_{IR}(s)]/D_1.$$

By solving this system of simultaneous equations, we obtain

$$\bar{A}V^0_{IR}(s) = [C_{11}(s)D_2 + C_{12}(s)C_6(s)]/[D_1D_2 - C_6(s)C_8(s)]$$

$$\bar{A}V^6_{IR}(s) = [C_{11}(s)C_8(s) + D_1C_{12}(s)]/[D_1D_2 - C_6(s)C_8(s)],$$

which, upon inversion, will give back $AV^0_{IR}(t)$

$$\mu^0_{IR}(t) = \int_0^t AV^0_{IR}(u)\,du.$$

The LT of $\mu^0_{IR}(t)$ is

$$\tilde{\mu}^0_{IR}(s) = \bar{A}V^0_{IR}(s)/s,$$

which, upon inversion, will give $\mu^0_{IR}(t)$.

Expected busy period of the server due to repair work of type j [j = 1 *to* s] *carried out in* [0, t]. We have

$$AV^0_{R(j)}(t) = AV^0_{7(j)}(t) + AV^0_{10(j)}(t)$$

$$AV^0_{R(j)}(t) = [a_1(t) * AV^1_{R(j)}(t)] + [a_2(t) * AV^4_{R(j)}(t)] + [a_3(t) * AV^9_{R(j)}(t)]$$

$$AV^1_{R(j)}(t) = \sum_{i=1}^{r} p_i[h(t) * AV^{2(i)}_{R(j)}(t)]$$

$$AV^{2(i)}_{R(j)}(t) = k_i(t) * AV^0_{R(j)}(t)$$

$$AV^4_{R(j)}(t) = \sum_{i=1}^{r} p_i[h(t) * AV^{5(i)}_{R(j)}(t)]$$

$$AV^{5(i)}_{R(j)}(t) = k_i(t) * AV^6_{R(j)}(t)$$

$$AV^6_{R(j)}(t) = \sum_{j'=1}^{s} \int_0^t b_1^{j'}(u)\phi^0_{R(j)}(u, t-u)\,du + \sum_{j'=1}^{s} [a_6^{j'}(t) * AV^6_{R(j)}(t)] + \sum_{j'=1}^{s} [b_2^{j'}(t) * AV^{10(j')}_{R(j)}(t)] + q_j \int_0^t l(w)\bar{M}j(t-w)[1 - F(w)]\,dw,$$

where

$$\phi^0_{R(j)}(x, t) = [a_9(x, t) * AV^1_{R(j)}(t)] + [a_{10}(x, t) * AV^4_{R(j)}(t)] + [a_{11}(x, t) * AV^9_{R(j)}(t)]$$

$$AV^9_{R(j)}(t) = \sum_{j'=1}^{s} q_{j'}[l(t) * AV^{10(j')}_{R(j)}(t)]$$

$$AV^{10(j')}_{R(j)}(t) = \delta_{jj'}\bar{M}_j(t) + m_{j'}(t) * AV^6_{R(j)}(t).$$

Upon taking the Laplace transforms of these expressions and simplifying, we obtain,

$$\bar{A}V^0_{R(j)}(s) = [C_{13}(s) + C_6(s)\bar{A}V^6_{R(j)}(s)]/D_2$$

and

$$\bar{A}V^0_{R(j)}(s) = [C_{14}(s) + C_8(s)\bar{A}V^0_{R(j)}(s)]/D_1.$$

This system of simultaneous equations gives

$$\bar{A}V^0_{R(j)}(s) = [C_{13}(s)D_2 + C_{14}(s)C_6(s)]/[D_1D_2 - C_6(s)C_8(s)]$$

and

$$\bar{A}V^6_{R(j)}(s) = [C_{13}(s)C_8(s) + D_1C_{14}(s)]/[D_1D_2 - C_8(s)C_6(s)],$$

which, upon inversion, will give $AV^0_{R(j)}(t)$

$$\mu^0_{R(j)}(t) = \int_0^t AV^0_{R(j)}(u)\,du.$$

The LT of $\mu^0_{R(j)}(t)$ is

$$\tilde{\mu}^0_{R(j)}(s) = \bar{A}V^0_{R(j)}(s)/s,$$

which, upon inversion, will give back $\mu^0_{R(j)}(t)$.

4.2. *Cost analysis*

The total expected expenditure is obtained by considering the expected busy period of the server due to on-line inspection, on-line PM action of type i ($i = 1$ to r), inspection associated with repair and repair work of type j ($j = 1$ to s).

Let $E[C(t)]$ be the expected total cost incurred in $[0, t]$. Then

$$E[C(t)] = C_{OI}\mu^0_{OI}(t) + \sum_{i=1}^{r} C_{M(i)}\mu^0_{M(i)}(t) + C_{IR}\mu^0_{IR}(t) + \sum_{j=1}^{s} C_{R(j)}\mu^0_{R(j)}(t),$$

where C_{OI}, $C_{M(i)}$, C_{IR} and $C_{R(j)}$ are as defined earlier.

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