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Signal constellations employing multiplicative groups of Gaussian and Eisenstein integers for Enhanced Spatial Modulation

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ABSTRACT

In this paper, we propose two new signal constellation designs employing Gaussian and Eisenstein Integers for Enhanced Spatial Modulation (ESM). ESM is a novel technique which was propounded by Cheng et al. The advantage of ESM over other Spatial Modulation (SM) schemes lies in its ability to enhance spectral efficiency while keeping the energy efficiency intact. This is done by activating either one or two antennas judiciously depending upon the required trade-off. In ESM, information radiated from the antennas depends upon index of the active transmit antenna combination(s) and also on the set of constellation points chosen, which may include points from multiple constellations. In this paper, we propose signal constellations based on multiplicative groups of Gaussian and Eisenstein integers. The set comprising of Gaussian and Eisenstein integers serves as primary and secondary constellation points for Gaussian Enhanced Spatial Modulation (GESM) scheme. The secondary constellation points are deduced from a single geometric interpolation from the primary constellation points. The Monte Carlo simulation results indicate that the proposed nonuniform constellations achieve impressive SNR gains compared to conventional constellation points used in the design of ESM. This new design has been described for MIMO employing 4×4 and 8×8 antenna configurations with only two active antennas. Furthermore, a closed form expression for the pairwise error probability (PEP) for the GESM scheme has been deduced. The PEP is utilized to determine the upper bound on the average bit error probability (ABEP). Our simulations indicate that the proposed GESM from Gaussian and Eisenstein integers scheme outperforms all the other variants of SM including conventional ESM by at least 2.5 dB at an average bit error ratio (ABER) of 10^{-5}. Close correspondence between the theoretical analysis and the Monte Carlo simulation results are observed.

1. Introduction

Multiple Input Multiple Output (MIMO) transmission techniques are broadly utilized as a part of wireless communication frameworks and are an integral part of the next generation 5G wireless communication systems, due to their ability to provide enhanced spectral efficiency along with improved reliability [1]. The likelihood of restricted direct device-to-device (D2D) communication has been introduced as an augmentation with the 4G/LTE-A specifications. In the next generation 5G era, exhaustive use of D2D communication as a feature of the general wireless communication is envisaged. The goal of incorporating these innovations is to have improved information throughput between devices in close proximity. A spectral efficiency of almost 10 bps/Hz to 16 bps/Hz has to be achieved between base station (BS) and mobile station (MS) in every cell to meet the data transfer requirements pertaining to 5G standards [1,2]. One of the fundamental limitation for mobile device is its physical size and antenna placement. A minimum spectral efficiency of 10 bps/Hz has to be extracted from a 4×4 system and 16 bps/Hz through a 8×8 system. To satisfy this demand, modulation schemes proposed for 5G should possess high spectral efficiency and energy efficiency [2]. Unlike MIMO spatial multiplexing (SMX) techniques, primary work on SM, [3,4] considers activation of single RF chain (single antenna). In order to compensate the reduction in spectral efficiency due to single active antenna, additional data bits in the form of active antenna indices are used to communicate information. In order to combat the growing requirement for spectrally efficient systems, Younis et al. proposed the idea of Generalized Spatial Modulation (GNSM) [5,6]. In this technique the number of active antennas are two, resulting in improved spectral efficiency due to increased number of active antenna combinations. A number of such schemes have been

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proposed in literature by Younis et al. such as Generalized SM (GNSM) and Variable Generalized SM (VGSM) [7]. Space Time Block Codes for SM systems were designed to increase the reliability of MIMO system [8]. In following years several improvements have been executed which lead to development of SM schemes such as Improved Spatial Modulation (ISM) or Extended Spatial Modulation (EXSM) (2012), proposed by Luna Rivera et al. [9,10]. Enhanced Spatial Modulation (ESM) (2014) given by Cheng et al. [11–13] and Quadrature Spatial Modulation (QSM) (2014), introduced by Mesleh et al. Among these QSM is an extension to Mesleh's original SM scheme which concentrates on improving spectral efficiency [14].

ESM was designed by consolidating new concepts in MIMO based on individual trade offs. First and foremost was to transmit information through single active antenna combination from primary constellation points followed by two active antennas which employs secondary constellation points. For multi-stream SM transmission techniques a single geometric interpolation has been carried out to activate two antenna combinations all the time, that was designated as ESM type1 and ESM type2 [13]. The demand for lower energy consumption per transmitted symbol lead us to explore non-uniform signal constellations with minimum symbol energy and higher distance of separation, as compared to the conventional constellation points.

The primary contributions of this paper are specified as below:

- To propose two new signal designs for ESM technique (GESM) from Gaussian and Eisenstein integers which pave the way to increased spectral efficiency and performance with minimum symbol energy (Section 3).
- Determination of a tight upper bound on the performance of GESM scheme which is in excellent agreement with performance plots obtained by Monte Carlo simulation (Section 4).
- Performance analysis of high rate ESM scheme under conditions of Rayleigh fading (Section 5).

The organization of this paper is as follows: In Section 2, we describe a brief description of system and channel models. Proposed GESM for single stream and multistream SM systems is discussed in Section 3. An analytical approach (closed form expression for ABER under Rayleigh fading channels) for proposed GESM scheme is explained in Section 4, followed by Complexity analysis in Section 5. Monte Carlo simulation results and performance analysis is described in Section 6. The paper is concluded in Section 7 with a discussion of possible enhancements that can be carried out to meet the next generation (5G) standards.

2. System and channel model

Before examining the idea of proposed Gaussian and Eisenstein signal designs used in GESM, the notion of Gaussian integers, Eisenstein integers and other framework models are described, which are utilized as the basis for relative comparison.

2.1. Gaussian and Eisenstein–Jacobi integers

Gaussian integers are numbers from the complex field of the form $Z = a + bi$ and $q = 4K + 1 = a^2 + b^2, (K = 0, 1, 2, 3 \ldots)$. Here a, b, K are integers and q is a prime number. The Gaussian prime number is defined as $\Pi = a + bi$. The set of integers $\left\{z_i = i \; mod \; \Pi = i - [\frac{i\Pi^*}{\Pi\Pi^*}]\Pi; i = 0, 1 \cdots q-1\right\}$, where Π^* is the complex conjugate of Π, is a field isomorphic to Galois field $GF(K)$. In this paper, we define F_Π as field of Gaussian integers isomorphic to $GF(K)$ [15].

In [16], Huber defined a map called as the Eisenstein map for prime numbers of the form $q' = 6b + 1$ i.e. $q' = 7; 13; 19 \cdots$. Eisenstein–Jacobi (EJ) integer ω is a complex number of the form $\omega = u + \rho\beta$, here $\rho = (\frac{-1+i\sqrt{3}}{2})$, such that $q' = u^2 + 3\beta^2$ where u and β are integers. These primes are product of two conjugate Eisenstein–Jacobi integers ζ, ζ^* defined as $\zeta = u + \beta + \rho \cdot 2\beta$ and its conjugate $\zeta^* = u + \beta + \rho^2 \cdot 2\beta$. The set of integers $\left\{\zeta(i) = i \bmod \Pi \triangleq i - [(\frac{i\Pi^*}{\Pi\Pi^*})]\Pi \; for \; i = 0, 1, 2 \cdot\cdot.q' - 1\right\}$ form a field E_ζ isomorphic to $GF(q')$

In communication scenario the multiplicative groups $F_\Pi \setminus \{0\}$ and $E_\zeta \setminus \{0\}$ can be considered as a two dimensional signal constellations. In this paper, we explore the use of above signal constellations for ESM systems. The systems employing these are named as Gaussian ESM (GESM) systems (see Tables 1 and 2).

Table 1
q, Π, a and b values.

q	Π	a	b
5	2+i	−1	1+i
13	3+2i	−2	1+2i
17	4+i	−2	2+i

Table 2
q', Π, u *and* β values.

q'	Π	u	β
7	3+2ρ	2	1
13	3+4ρ	1	2
19	5+2ρ	4	1

2.2. Enhanced SM (ESM)

ESM scheme for single stream SM systems are designed to have maximum Euclidean distance between the points chosen for transmission. ESM schemes employ primary and secondary constellations (for example: 16 QAM and QPSK). Single antenna combinations are activated through the primary constellation and two antenna combinations are activated through secondary constellation. When secondary constellation points are radiated from two antennas then each radiated symbol possesses exactly half the energy of the symbol radiated from the primary constellation. The spectral efficiency provided by this scheme is quantified as shown below [13],

$$\eta = \log_2 \left(\binom{N_t}{1} \times P_c + 2 \binom{N_t}{2} \times S_c \right), \quad (1)$$

where P_c represents the number of points in primary constellation and S_c is the number of points in secondary constellation.

2.3. Quadrature SM (QSM)

A new methodology was proposed by Mesleh et al., which uses the aspects of in-phase and quadrature phase points of the constellation. These are transmitted independently from separate active antennas (1 or 2). This method is known as Quadrature Spatial Modulation (QSM). QSM technique produces an improvement in spectral efficiency as compared to conventional SM schemes. It exhibits a spectral efficiency as given below [14],

$$\eta = 2 \times \log_2(N_t) + \log_2(M). \quad (2)$$

2.4. Enhanced SM (ESM) type-1 and type-2:

ESM type-1,type-2 schemes are used to enhance the throughput of multistream SM system. The idea behind these schemes is to increase the number of active antenna combinations by initiating single step geometrical interpolation in the signal constellation plane. This new designed constellation points possess higher euclidean distance and are involved in reducing the total transmit energy [13].

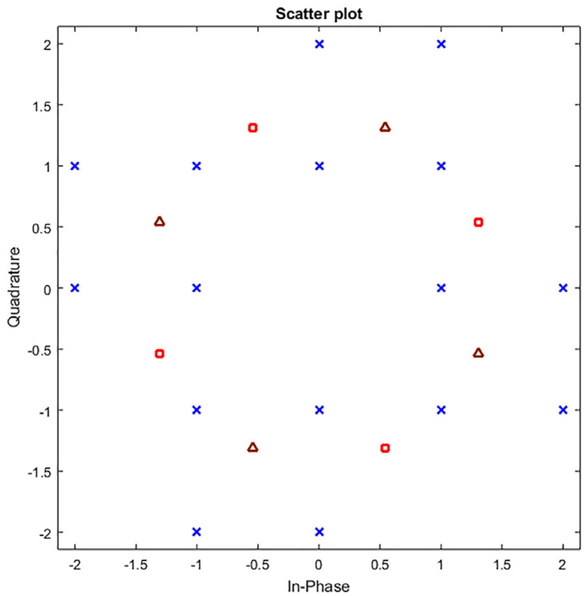

Fig. 1. Gaussian constellations used for 4×4 MIMO system yielding $\eta = 8$ bps/Hz: The blue crosses represent $\{GF(17) \setminus \{0\}, \cdot\}$, the red square and brown triangle represents the rotated $\{GF(5) \setminus \{0\}, \cdot\}$ signal constellations.

3. Proposed enhanced SM (GESM) design from Gaussian and Eisenstein integers:

We consider a MIMO system operating on Rayleigh fading channel, then the received signal is denoted by:

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{3}$$

where $\mathbf{H}$ is a $N_r \times N_t$ channel matrix, N_r is the number of receive antennas, N_t is the number of transmit antennas, $\mathbf{x}$ is a transmitted vector which possess normalized power of $\mathbf{x} = \frac{\mathbf{s}}{\sqrt{\mathbf{E_s}}}$, vector $\mathbf{x}$ is transmitted over a frequency flat MIMO channel of size $N_t \times 1$, $\mathbf{n}$ is a circularly symmetric complex Gaussian noise and a column vector which is denoted as $C\mathcal{N}(0, \sigma^2)$ independent and identically distributed (i.i.d).

Similar to conventional ESM, the proposed GESM scheme activates one antenna for radiating symbol from primary constellation and activates two antennas while communicating symbols from secondary constellation in order to achieve the required throughput (bpcu) this has been displayed in Fig. 1. A similar mapping for GSM systems over Eisenstein integers is shown in [17].

$$\mathbf{x} \in \{\text{Ł}1, \text{Ł}2, \text{Ł}3\}. \tag{4}$$

In the proposed design, transmitted codeword vector $\mathbf{x}$ is given by (4) and it can be inferred that $\{\text{Ł}1, \text{Ł}2, \text{Ł}3\}$ is the set of transmission vectors chosen to yield a spectral efficiency of 8 bps/Hz for a 4×4 MIMO system. Here, $x1$ refers to (blue cross) constellation points obtained from the multiplicative group $\{GF(17) \setminus \{0\}, \cdot\}$. A detailed explanation is given below.

$$\text{Ł}1 = \left\{ \begin{bmatrix} x1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ x1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ x1 \end{bmatrix} \right\} \tag{5}$$

$$\text{Ł}2 = \left\{ \begin{bmatrix} x2 \\ x2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} x2 \\ 0 \\ x2 \\ 0 \end{bmatrix}, \begin{bmatrix} x2 \\ 0 \\ 0 \\ x2 \end{bmatrix}, \begin{bmatrix} 0 \\ x2 \\ x2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x2 \\ 0 \\ x2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ x2 \\ x2 \end{bmatrix} \right\} \tag{6}$$

$$\text{Ł}3 = \left\{ \begin{bmatrix} x3 \\ x3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} x3 \\ 0 \\ x3 \\ 0 \end{bmatrix}, \begin{bmatrix} x3 \\ 0 \\ 0 \\ x3 \end{bmatrix}, \begin{bmatrix} 0 \\ x3 \\ x3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x3 \\ 0 \\ x3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ x3 \\ x3 \end{bmatrix} \right\} \tag{7}$$

$$x1 = \left\{ \begin{matrix} 1, & 1+1i, & 2i-1, & -2i, & 1i, & -1+1i, \\ -2, & 2-1i, & -1, & -1-1i, & -2i, & 1+2i, \\ & -1i, & 1-1i, & 2, & -2+1i & \end{matrix} \right\} \tag{8}$$

$$x2 = \left\{ \begin{matrix} -1.3066 - 0.5412i, & 0.5412 - 1.3066i, \\ -0.5412 + 1.3066i, & 1.3066 + 0.5412i \end{matrix} \right\} \tag{9}$$

$$x3 = \left\{ \begin{matrix} -1.3066 + 0.5412i, & -0.5412 - 1.3066i, \\ 0.5412 + 1.3066i, & 1.3066 - 0.5412i \end{matrix} \right\}. \tag{10}$$

It is observed that, if we increase the number of codeword vectors by including set Ł4 the spectral efficiency increases to 8.5 bps/Hz.

$$\text{Ł}4 = \left\{ \begin{bmatrix} x2 \\ x3 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} x2 \\ 0 \\ x3 \\ 0 \end{bmatrix}, \begin{bmatrix} x2 \\ 0 \\ 0 \\ x3 \end{bmatrix}, \begin{bmatrix} 0 \\ x2 \\ x3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ x2 \\ 0 \\ x3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ x2 \\ x3 \end{bmatrix} \right\}. \tag{11}$$

$x2$ refers to (Red circle) rotated constellation points obtained from the multiplicative group $\{GF(5) \setminus \{0\}, \cdot\}$. The angle of rotation of $x2$ is chosen to be 22.5°. $x3$ refers to (Brown Triangle) rotated constellation points obtained from the multiplicative group $\{GF(5) \setminus \{0\}, \cdot\}$. The angle of rotation is chosen such that angular difference between points of $x3$ and corresponding points of $x2$ is 45° which leads to maximization of the Euclidean distance.

The second most important aspect in designing this constellation is the average energy required to transmit the codeword. The average codeword energy of a conventional ESM system which employs a 4×4 system with (16 QAM, QPSK1, QPSK2) primary and secondary constellation points to produce a spectral efficiency of 8 bps/Hz is given by

$$E_{\text{avg}}(ESM) = 10 + 1 + 1 = 12. \tag{12}$$

While the average codeword energy of the proposed GESM which employs a 4×4 system with $(\{GF(17) \setminus \{0\}, \cdot\}, \{GF(5) \setminus \{0\}, \cdot\}, \{GF(5) \setminus \{0\}, \cdot\})$ primary and secondary constellation points to produce a spectral efficiency of 8 bps/Hz is as below

$$E_{\text{avg}}(GESM) = 3 + 2.0001 + 2.0001 = 7.0002. \tag{13}$$

The above observation indicates that the average energy per transmitted codeword required in the proposed scheme is $\sim 42\%$ less than the energy required for the conventional ESM scheme.

Note: The euclidean distance between two codeword vectors is calculated using [12]

$$\ell^2_{min} = \min \left\| X - X' \right\|^2. \tag{14}$$

It can be observed that the Euclidean distance between conventional ESM and proposed GESM constellation points is 0.5518. However, from (14) the minimum Euclidean distance between codeword vectors is observed to be 42.03 in the proposed GESM scheme while Euclidean distance is 23.03 for the conventional ESM scheme. This results in an improved BER performance for the proposed GESM scheme.

Since the spectral efficiency is directly proportional to the number of codeword vectors, an increase in the size of the codebook increases the spectral efficiency. To achieve a spectral efficiency of 9.5 bps/Hz in a 4×4 MIMO system, we have considered $x2$ as Eisenstein multiplicative group $\{GF(7) \setminus 0, \cdot\}$ and $x3 \subset x1$ as shown below. The signal constellation for $\eta = 9.5$ bps/Hz is shown

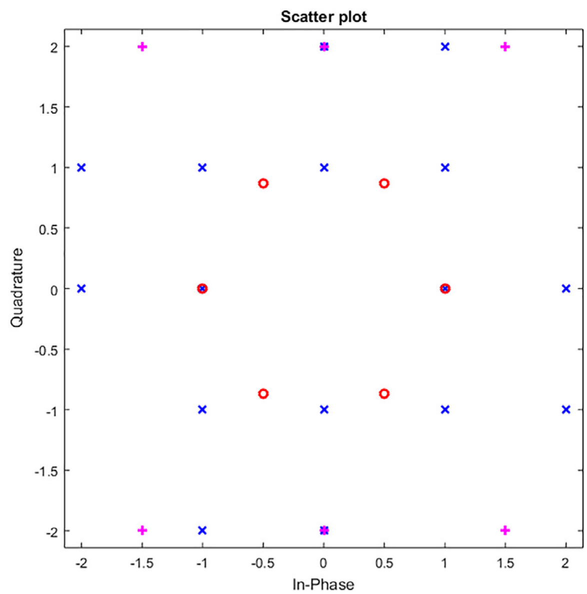

Fig. 2. Gaussian constellations used for 4×4 MIMO system yielding $\eta = 9.5$ bps/Hz: The blue crosses represent $\{GF(17)\setminus\{0\},\cdot\}$, the red circle represents Eisenstein $\{GF(7)\setminus\{0\},\cdot\}$ and magenta plus symbol indicates the subset of $\{GF(17)\setminus\{0\},\cdot\}$ constellations.

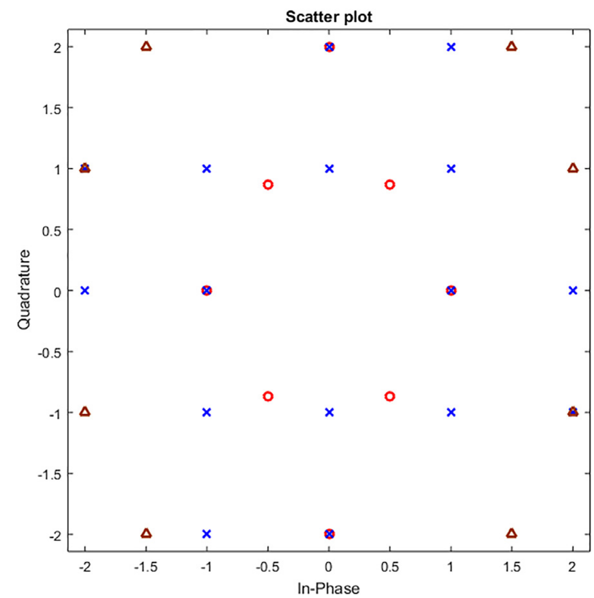

Fig. 3. Gaussian constellations used for 4×4 MIMO systems yielding $\eta = 10.3$ bps/Hz: The blue crosses represent $\{GF(17)\setminus\{0\},\cdot\}$, the red circle represents Eisenstein $\{GF(7)\setminus\{0\},\cdot\}$ with first interpolated points and brown triangle symbol indicates the subset of $\{GF(17)\setminus\{0\},\cdot\}$ constellations.

in Fig. 2.

$$\begin{aligned}\rho &= (-1+\sqrt{3}i)/2\\ x2 &= \{1,\ 1+\rho,\ \rho,\ -1,\ -1-\rho,\ -\rho\}\end{aligned} \tag{15}$$

$$x3 = \{1.5-2i,\ 1.5+2i,\ -1.5-2i,\ -1.5+2i,\ 2i,\ -2i\}. \tag{16}$$

Similarly, to achieve a spectral efficiency of 10.3 bps/Hz in a 4×4 MIMO system, we have constructed $x2$ by interpolating the Eisenstein multiplicative group $\{GF(7)\setminus\{0\},\cdot\}$ and $x3 \subset x1$ as shown below.

$$\begin{aligned}\rho &= (-1+\sqrt{3}i)/2\\ x2 &= \{1,\ 1+\rho,\ -2i,\ \rho,\ -1,\ 2i,\ -1-\rho,\ -\rho\}\end{aligned} \tag{17}$$

$$\begin{aligned}x3 = \{&1.5-2i,\ 1.5+2i,\ -1.5-2i,\ -1.5+2i,\ 2+1i,\\ &2-1i,\ -2+1i,\ -2-1i\}.\end{aligned} \tag{18}$$

The signal constellation for $\eta = 10.3$ bps/Hz is shown in Fig. 3. In general, spectral efficiency is given by

$$\eta = \log_2\left(\binom{N_t}{1} \times P_c + 3\binom{N_t}{2} \times S_c\right) \tag{19}$$

where P_c is the number of points in primary constellation $(F_{4+i}\setminus\{0\})$ and S_c is the number of points in secondary constellation $\left(F_{\Pi}\setminus\{0\},\ G_{\varsigma}\setminus\{0\}\right)$. It is to be noted that η can be increased by employing higher order multiplicative groups of Gaussian and Eisenstein integers. Furthermore, the proposed scheme can be generalized for any number of transmit antennas. Simulation results for constructions over 8×8 MIMO system is shown in Section 4.

3.1. GESM Generalization to multistream SM systems

Basic concept of MSM with ML decoding analysis was primarily discussed in [18]. In this section, we propose an extended version of GESM to Multistream SM systems. Considering the same number of signal constellation points as given in Eqs. (8)–(10) the extended scheme for a 4×4 MIMO systems, is described as below

$$\mathbf{x} \in \{\text{Ł}1,\ \text{Ł}2,\ \text{Ł}3\} \tag{20}$$

$$\text{Ł}1 = \left\{\begin{bmatrix}x11\\x12\\0\\0\end{bmatrix},\begin{bmatrix}x11\\0\\x12\\0\end{bmatrix},\begin{bmatrix}x11\\0\\0\\x12\end{bmatrix},\begin{bmatrix}0\\x11\\x12\\0\end{bmatrix},\begin{bmatrix}0\\x11\\0\\x12\end{bmatrix},\begin{bmatrix}0\\0\\x11\\x12\end{bmatrix}\right\} \tag{21}$$

$$\text{Ł}2 = \left\{\begin{bmatrix}x2\\x2\\0\\0\end{bmatrix},\begin{bmatrix}x2\\0\\x2\\0\end{bmatrix},\begin{bmatrix}x2\\0\\0\\x2\end{bmatrix},\begin{bmatrix}0\\x2\\x2\\0\end{bmatrix},\begin{bmatrix}0\\x2\\0\\x2\end{bmatrix},\begin{bmatrix}0\\0\\x2\\x2\end{bmatrix}\right\} \tag{22}$$

$$\text{Ł}3 = \left\{\begin{bmatrix}x3\\x3\\0\\0\end{bmatrix},\begin{bmatrix}x3\\0\\x3\\0\end{bmatrix},\begin{bmatrix}x3\\0\\0\\x3\end{bmatrix},\begin{bmatrix}0\\x3\\x3\\0\end{bmatrix},\begin{bmatrix}0\\x3\\0\\x3\end{bmatrix},\begin{bmatrix}0\\0\\x3\\x3\end{bmatrix}\right\} \tag{23}$$

$$\text{Ł}4 = \left\{\begin{bmatrix}x2\\x3\\0\\0\end{bmatrix},\begin{bmatrix}x2\\0\\x3\\0\end{bmatrix},\begin{bmatrix}x2\\0\\0\\x3\end{bmatrix},\begin{bmatrix}0\\x2\\x3\\0\end{bmatrix},\begin{bmatrix}0\\x2\\0\\x3\end{bmatrix},\begin{bmatrix}0\\0\\x2\\x3\end{bmatrix}\right\} \tag{24}$$

$$x11 = \left\{\begin{matrix}1, & 1+1i, & 1i, & -1+1i,\\ -1, & -1-1i, & -1i, & 1-1i,\end{matrix}\right\} \tag{25}$$

$$x12 = \left\{\begin{matrix}2i-1, & -2i, & -2, & 2-1i,\\ -2i, & 1+2i, & 2, & -2+1i\end{matrix}\right\} \tag{26}$$

$$x2 = \left\{\begin{matrix}-1.3066-0.5412i, & 0.5412-1.3066i,\\ -0.5412+1.3066i, & 1.3066+0.5412i\end{matrix}\right\} \tag{27}$$

$$x3 = \left\{\begin{matrix}-1.3066+0.5412i, & -0.5412-1.3066i,\\ 0.5412+1.3066i, & 1.3066-0.5412i\end{matrix}\right\}. \tag{28}$$

From the above analysis it can be noted that the spectral efficiency has been increased to 9.4 bps/Hz. That is, in general for a multistream SM system the spectral efficiency can be enhanced by increasing the number of codeword vectors. The generalization defined above for multistream systems results in a spectral efficiency given by

$$\eta = \log_2\left(|\text{Ł}_1| + |\text{Ł}_2| + |\text{Ł}_3| + |\text{Ł}_4|\right). \tag{29}$$

Note: Since the number of signal constellation points (primary and secondary) remain same, the average energy is same as given

Table 3
Minimum Euclidean distance of ESM and GESM.

η	ESM	GESM
8 bpcu(4 × 4)	23.03	43.03
10 bpcu(4 × 4)	45	57
12 bpcu(8 × 8)	92	122

Table 4
Maximum symbol energy of ESM and GESM.

η	ESM	GESM
8 bpcu(4 × 4)	18	5
10 bpcu(4 × 4)	18	5
12 bpcu(8 × 8)	18	5

in (12) and (13), the minimum Euclidean distance between codeword vectors is given in following Tables.

Table 3 and Table 4 show the comparison between conventional ESM and GESM in terms of minimum Euclidean distance between two codeword vectors and maximum energy consumption per symbol. It can be inferred that the maximum symbol energy reduces with increase in $\ell_{\min}$, this is due to the fact that constellations chosen have same lower energy points as that of conventional ESM constellation. Moreover, points representing higher energy are different.

3.2. Quasi static Rayleigh fading channels

In this work the following frame structure is considered to estimate the channel. Channel estimation is performed for every 1000 data symbols to obtain the optimal channel realization [7,19]. Further, a complete procedure for channel estimation and realization is given by author in [19].

The frame structure explained here is specifically to a $4 \times N_r$ system, but can be generalized to any $N_t \times N_r$ system by considering N_t pilots. To estimate the channel coefficients individually from the respective transmit antennas, we have made use of a transmit frame length comprising of 1202 symbols. The 160 symbols out of 1202 are used for frame synchronization and the next 1024 symbols (pilot symbols) are used to estimate the channel behavior. Peak detection technique is employed to achieve synchronization. The total of 1024 symbols in the transmit frame of which first 256 symbols constitute to pilot 1 are transmitted from first active transmit antenna, during this period all the remaining transmit antennas are deactivated. Similarly pilot 2, 3 and 4 are transmitted from active antennas 2, 3 and 4 respectively. In order to distinguish between two frames 18 zero valued symbols are padded at the start of every frame (see Fig. 4).

Following [7,20], realization of any fading channel is given by channel matrix.

$$\mathbf{H} = \check{\mathbf{H}} + \dot{\mathbf{H}} \tag{30}$$

where $\check{\mathbf{H}}$ is the mean matrix and $\dot{\mathbf{H}}$ is a $N_r \times N_t$ channel matrix whose entries are i.i.d complex gaussian random numbers with zero mean and unit variance ($\mathcal{CN}(0, 1)$). The mean matrix for a Rayleigh fading channel can be evaluated [21] as $\check{\mathbf{H}} = \mathbf{0}_{N_r \times N_t}$ matrix [7]. Optimal ML detection strategy is employed for the considered GESM scheme. Joint estimation of the antenna indices and M-QAM information symbols are estimated as $\log_2\left(\binom{N_t}{1} \times P_c + 3\binom{N_t}{2} \times S_c\right)$ bits.

$$(\hat{\iota}, \hat{x}) = \underset{X \in \{Ł1, Ł2, Ł3, Ł4\}}{\arg\min} \quad \|y - \mathbf{H}x\|_F^2 \tag{31}$$

where $\|\bullet\|_F$ is the Frobenius norm.

4. Analytical treatment for proposed GESM :

A mathematical analysis for the ABEP of the proposed GESM scheme is presented in this section. A quasi static Rayleigh fading environment is considered for our analysis. The conditional pairwise error probability (CPEP) for erroneous detection of $\hat{x}$ (when $\mathbf{x}$ is transmitted and $(\hat{x} \neq x)$) is calculated as given below, [12,13].

$$PEP(\mathbf{x} \to \widehat{\mathbf{x}}|\mathbf{H}) = \mathbf{P_r}(\|\mathbf{y} - \mathbf{Hx}\|_\mathbf{F}^2 > \|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\|_\mathbf{F}^2|\mathbf{H})$$
$$= P_r(\|\mathbf{Hx}\|^2 - \left\|\mathbf{H}\hat{\mathbf{x}}\right\|^2 - 2\Re\{\mathbf{y^H}(\mathbf{Hx} - \mathbf{H}\hat{\mathbf{x}})\} > \mathbf{0})$$

$$P(\mathbf{x} \longrightarrow \widehat{\mathbf{x}}|\mathbf{H}) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(\frac{\|\mathbf{H}(\mathbf{x} - \widehat{\mathbf{x}})\|^2}{4N_0 \sin^2\theta}\right) d\theta \tag{32}$$

$$P(\mathbf{x} \longrightarrow \widehat{\mathbf{x}}) = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + \frac{\|\mathbf{x} - \widehat{\mathbf{x}}\|^2}{4N_0}}\right)^{N_r} d\theta. \tag{33}$$

Following [22,19], The closed form expression for the PEP over quasi static Rayleigh fading channel can be given as

$$P(\mathbf{x} \longrightarrow \widehat{\mathbf{x}}) = \left(\frac{1 - \mu(c)}{2}\right) \sum_{k=0}^{N_r - 1} \binom{k}{N_r - 1 + k} \left(\frac{1 + \mu(c)}{4}\right)^k \tag{34}$$

where

$$\mu(c) \triangleq \left(\frac{c_{Rayleigh}}{1 + c_{Rayleigh}}\right) \quad c_{Rayleigh} = \frac{\|\mathbf{x} - \widehat{\mathbf{x}}\|^2}{4N_0}. \tag{35}$$

5. Computational complexity analysis

Following [13], receiver computational complexity analysis has been calculated in this section. We outline the receiver complexity as the number of floating point operations (flops) needed per ML decision metric. As given in [23,24], we consider every addition, subtraction, multiplication, division, and square-root operation as a single flop. From Eq. (3), ML decoder has to comprehensively search the space and need to compute 2^η decision metrics, here η is the spectral efficiency of a system. The computational complexity in terms of number of flops is given by:

C*ase*.1 computing $\mathbf{Hx}$: Consider $x \in Ł1$, $\mathbf{y} - \mathbf{Hx}$ in Eq. (31) simplifies to $\mathbf{y} - \mathbf{h_i}x_j$ where, $i \in [1, 2, \ldots, N_t]$ and $j \in [1, 2, \ldots, q]$. Evaluating this requires N_r multiplications/antenna/symbol, it has zero additions since only one antenna is active. Consider $x \in Ł2$, $\mathbf{y} - \mathbf{Hx}$ in Eq. (31) reduces to $\mathbf{y} - \mathbf{h_i}x_j - \mathbf{h_k}x_l$, where $i, k \in [1, 2, \ldots, N_t]$ and $j, l \in [1, 2, \ldots, q]$, $\mathbf{h_i}, \mathbf{h_k} \to N_r \times 1$ column vector $\in \mathbf{H}$. Following [13], $\mathbf{h_i}x_j - \mathbf{h_k}x_l$ is evaluated by computing all possible $\mathbf{h}x$. This results in $N_t \cdot N_r \cdot S_c$ multiplications, $\binom{N_t}{2} \cdot N_r \cdot S_c^2$ additions. Similarly if $x \in Ł3$, the total number of multiplications and additions required are similar to the one calculated for Ł2. The total computations required for $\mathbf{Hx}$ is given by

$$N_t \cdot S_p \cdot N_r + 2\left[N_t \cdot S_c \cdot N_r\right] + 2\left[\binom{N_t}{2} \cdot N_r \cdot S_c^2\right] \tag{36}$$

where, S_p is the cardinality of the primary constellation, S_c represents the cardinality of the secondary constellations.

C*ase*.2 computing $\mathbf{y} - \mathbf{Hx}$: The number of flops (subtractions) required are $2^\eta \cdot N_r$.

C*ase*.3 computing $\|\mathbf{y} - \mathbf{Hx}\|_\mathbf{F}^2$: This requires N_r complex multiplications and $N_r - 1$ additions per one possible x hence, the total number of flops required are $2^\eta \cdot (2N_r - 1)$.

Finally, computational complexity for the proposed GESM can be written as

$$N_t \cdot S_p \cdot N_r + 2\left[N_t \cdot S_c \cdot N_r\right] + 2^\eta \cdot N_r$$
$$+ 2\left[\binom{N_t}{2} \cdot N_r \cdot S_c^2\right] + 2^\eta \cdot N_r + 2^\eta \cdot (2N_r - 1). \tag{37}$$

18 zero valued symbols	160 synchronization pulses	**Pilot -1 256 symbols**	**Pilot -2 256 symbols**	**Pilot -3 256 symbols**	**Pilot -4 256 symbols**

Fig. 4. Channel estimation.

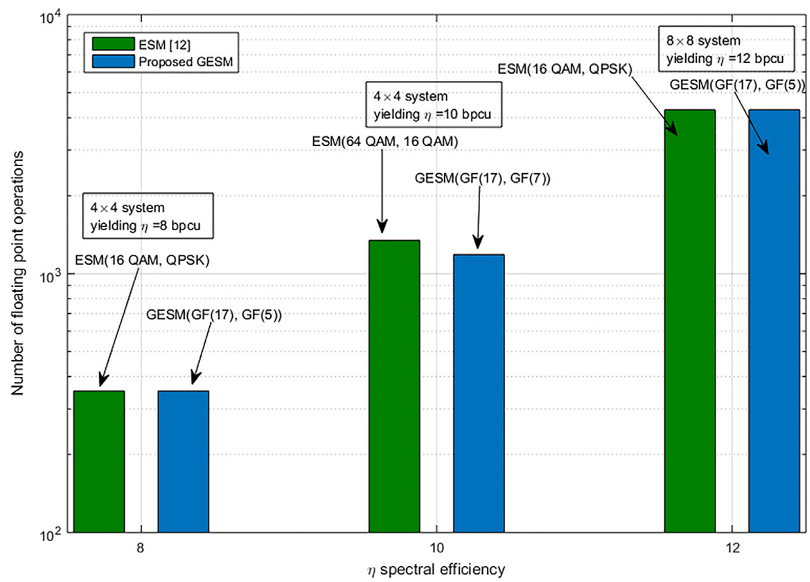

Fig. 5. Computational complexity of the proposed GESM with ESM.

Fig. 5, shows the computational complexity of the proposed GESM and ESM schemes for various spectral efficiencies. From Fig. 5, it is to be noted that for $\eta = 8, 12$ bpcu the complexity of the proposed GESM scheme is same as that of ESM scheme, this is due to the fact that the cardinality of primary and secondary constellations are same for both GESM and ESM schemes. However for the case $\eta = 10$ bpcu, the conventional ESM require 64 QAM and 16 QAM as the primary and secondary constellations whereas GESM requires GF(17) as the primary and GF(7) as the secondary constellation resulting in complexity reduction of 11.9%. For $N_t \times 1$ system the computational complexity can be obtained as given below:

To evaluate Eq. (31) the number of complex and real multiplications required can be described with a following example. For a 4 transmit MIMO system with $\eta = 8$ bpcu, GESM ML decoder needs to compute $w_{ij} = y - h_i x_j$ for three sets of antenna/constellation combinations, where h_i denote the ith column of H matrix. $x_j \in F_{4+i} \setminus \{0\}$. First set ℓ_1 comprises of four possible antenna combinations hence computation of w_{ij} results in 64 complex multiplications equivalently 256 real multiplications. Second set ℓ_2 having two active antenna combinations has w_{ij} given by $w_{ij} = -h_{i1}x_{j1} - h_{i2}x_{j2}$. Since $x_2 \in F_{1+2i} \setminus \{0\}$, the number of complex multiplication involved are $4 \times 4 = 16$. Similarly, ℓ_3 results in 16 complex multiplications. Further, computation for the squared modulus of each one of the w_{ij} terms, requires another 256 complex multiplications. Therefore, the total number of complex multiplications required in estimating 1 possible transmitted vector is given by 64+16+16+256=352. One complex multiplication is equivalent to 4 real multiplications hence total number of real multiplications required are 352 × 4 =1408. From the above analysis it is observed that for a 4 transmit 8 bpcu GESM framework, the computational complexity remains same as that of conventional ESM scheme. Extending the above analysis for $\eta = 10$ and 12 bpcu the computational complexity values are tabulated in Table 5.

Table 5
Receiver complexity for $N_r = 1$.

Spectral efficiency	8 bpcu(4×4)	10 bpcu(4×4)	12 bpcu(8×8)
ESM	352	1344	4288
GESM	352	1184	4288

6. Simulation results and observations:

In this section we have compared the performance of the proposed GESM scheme with the variants of SM (ESM,QSM,SM). We additionally assume perfect channel state information available at the receiver [25,26]. Simulations were performed for three spectral efficiencies $\eta = 8, 10, 12$ bps/Hz. To achieve a spectral efficiency of η =8 bps/Hz, the proposed GESM employs a 4×4 MIMO system. Further, to achieve a $\eta = 12$ bps/Hz a 8×8 MIMO system is considered. A minimum of 10^6 channel realizations have been considered for the estimation of ABER. To justify the effectiveness, these values are compared with the derived mathematical upper bound. Monte Carlo simulation method is used to substantiate the analytical BER performance of the proposed GESM scheme. A close correlation is observed between simulation and theoretic results.

In Fig. 6, we have shown the BER performance of a proposed GESM scheme with the variants of SM (QSM, ESM and SM) for a system employing 4×4 MIMO arrangement producing a spectral efficiency $\eta = 8$ bps/Hz over a quasi static flat Rayleigh fading channel. Proposed GESM uses $F_{4+i} \setminus \{0\}$ as primary constellation, $e^{j\theta_1}F_{2+i} \setminus \{0\}$ and $e^{j\theta_2}F_{2+i} \setminus \{0\}$ as the two secondary constellation points ($\theta_1 = 22.5, \theta_2 = 67.5$), conventional ESM uses 16 QAM and QPSK1, QPSK2 as the primary and secondary constellation points, QSM uses 16 QAM while SM uses 64 QAM. It is observed that the proposed GESM system outperforms conventional ESM, QSM and SM systems by approximately 2 dB, 2.5 dB and 5 dB respectively. Further it is observed that simulation results are in

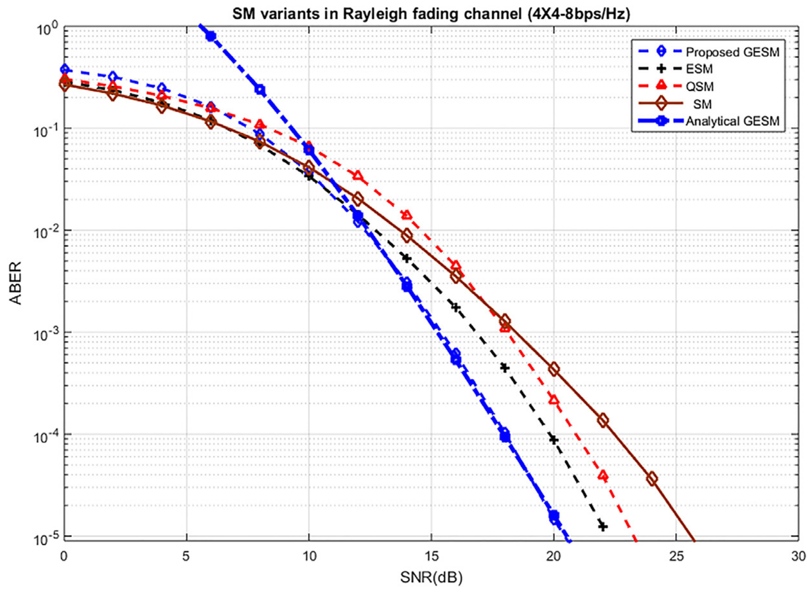

Fig. 6. BER performance analysis of GESM and variants of SM schemes in a 4 × 4 MIMO system yielding $\eta = 8$ bps/Hz.

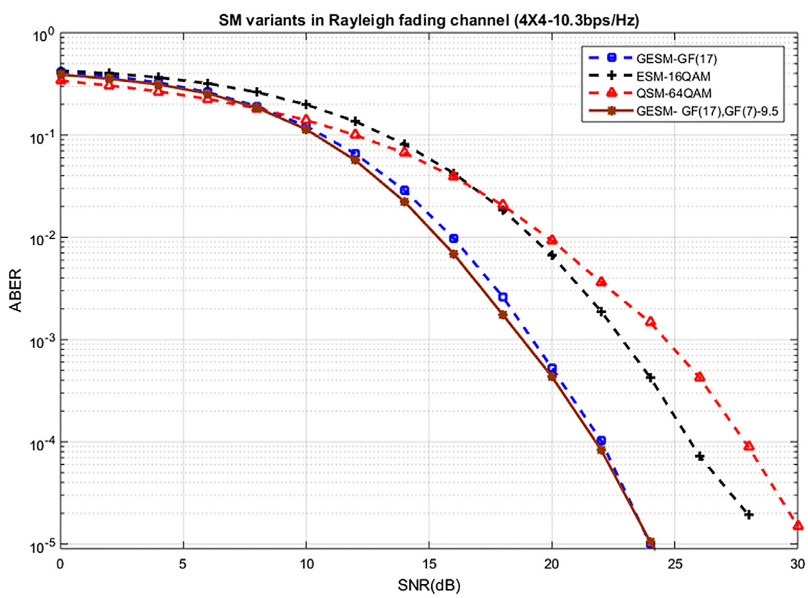

Fig. 7. BER performance analysis of GESM and variants of SM schemes in a 4 × 4 MIMO system yielding $\eta = 10.3$ bps/Hz. (Exception: GESM brown line, represents secondary constellation extension producing 9.5 bps/Hz).

close correspondence with analytical upper bound at SNRs greater than 10 dB (Asymptotic values).

Fig. 7, shows two important observations: First, GESM scheme employing $e^{j\theta_1}F_{2+i} \setminus \{0\}$ and $e^{j\theta_2}F_{2+i} \setminus \{0\}$ as the two secondary constellations can be extended to yield a spectral efficiency of 9.5 bps/Hz that is marked in brown line. Second: If we choose GESM scheme with $F_{4+i} \setminus \{0\}$, $F_{3+2\rho} \setminus \{0\}$ and geometric interpolation of points pertaining to $F_{3+2\rho} \setminus \{0\}$, a spectral efficiency of 10.3 bps/Hz is achieved, while the conventional ESM uses 64 QAM and two 16 QAMs as the primary and secondary constellation points, QSM uses 64 QAM. It is observed that the proposed GESM system has an improvement of approximately 4 dB and 5 dB over conventional ESM and QSM systems respectively.

Fig. 8. gives the performance analysis of GESM and other variants of SM schemes in a 8 × 8 MIMO arrangement producing a spectral efficiency $\eta = 12.5$ bps/Hz over a quasi static flat Rayleigh fading channel. Proposed GESM uses $F_{4+i} \setminus \{0\}$ as primary constellation, $e^{j\theta_1}F_{2+i} \setminus \{0\}$ and $e^{j\theta_2}F_{2+i} \setminus \{0\}$ as the two secondary constellation points ($\theta_1 = 22.5, \theta_2 = 67.5$), conventional ESM uses 16 QAM and QPSK1, QPSK2 as the primary and secondary constellation points and QSM uses 16 QAM to produce 12 bps/Hz. It is observed that the proposed GESM system is superior to conventional ESM and QSM systems by approximately 4 dB and 6 dB respectively. Furthermore, this GESM scheme can be extended to multistream SM systems as discussed previously, is demonstrated in Fig. 7.

Fig. 9, shows the comparison of maximum achievable spectral efficiency with variable number of antennas. It is observed that for $N_t = 4$ the proposed GESM scheme achieves a average spectral efficiency of 8.5 bps/Hz, Multistream GESM scheme attains spectral

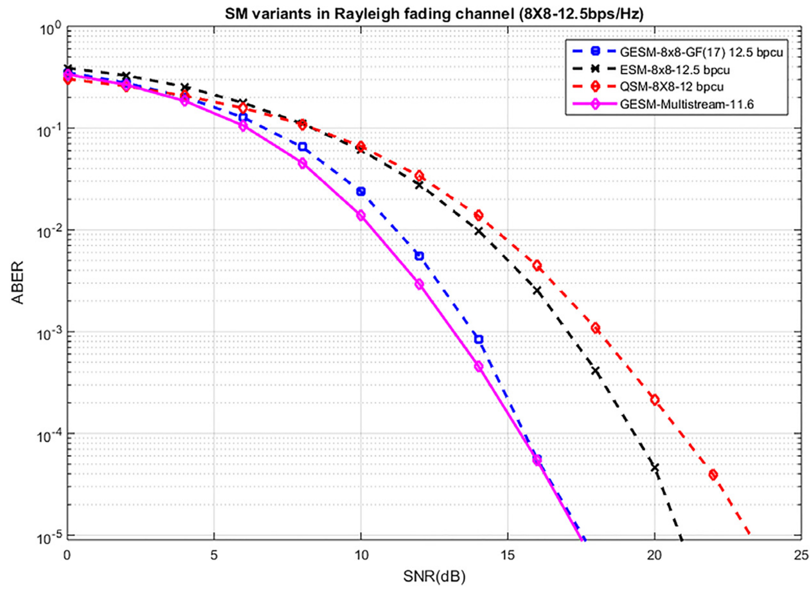

Fig. 8. BER performance analysis of GESM and variants of SM schemes in a 8 × 8 MIMO systems yielding $\eta = 12.5$ bps/Hz. (Exception: GESM magenta line, represents secondary constellation extension producing 11.3 bps/Hz).

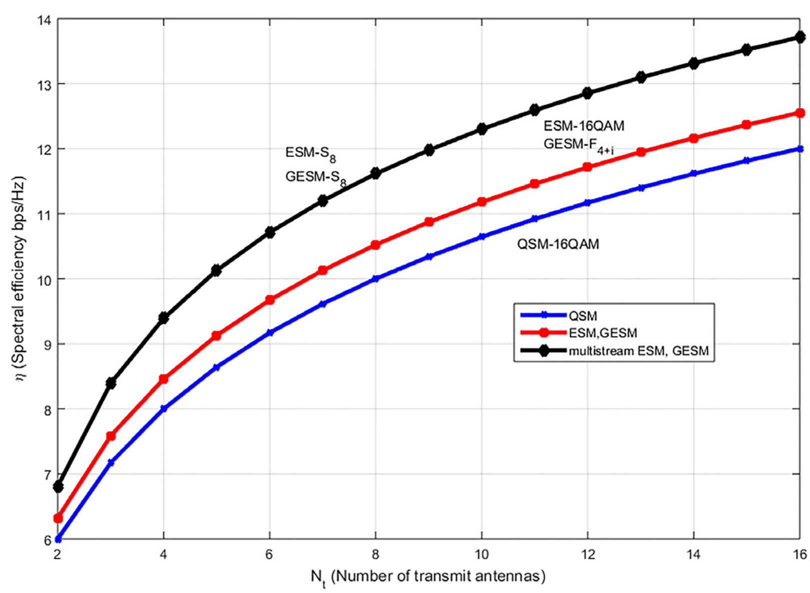

Fig. 9. A comparison of achievable spectral efficiencies with variable number of transmit antennas, N_t, and with M = 16-QAM and F_{4+i} modulation for various SM systems.

efficiency of 10.3 bps/Hz. QSM system achieves spectral efficiency of 8 bps/Hz. Note that all these schemes activate either one or two transmit antennas only.

7. Conclusions and future work

In this paper, we have investigated signal constellations for ESM systems from multiplicative groups of Gaussian and Eisenstein–Jacobi integers and named it as GESM. This scheme can be generalized to single stream SM as well as multistream SM configurations. Moreover, extension of this construction and mapping to higher antenna configurations has been devised and analyzed. The derived mathematical upper bound and Monte Carlo simulation results show that proposed new constellation design for ESM exhibits remarkable improvement in SNR gains (approximately 2.5 dB) compared to traditional ESM systems employing standard constellation points. Since the signal points are not a power of two, fractional bps/Hz values are obtained in our Monte Carlo simulations. In conclusion, it is observed that the new GESM scheme is energy efficient as well as spectrally efficient and this scheme is well suited to meet the requirements of LTE-Advanced and any 5G wireless systems employing MIMO architecture that may evolve in the future.

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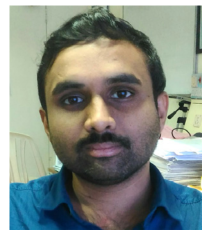

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