FISEVIER

Contents lists available at ScienceDirect

# **Applied Thermal Engineering**

journal homepage: www.elsevier.com/locate/apthermeng



# Theoretical analysis of the effect of water and ethanol injection on axial compressor instabilities

Pallav Kant<sup>a</sup>, B.S. Kishore Krishna<sup>b</sup>, A.M. Pradeep<sup>c,\*</sup>, Sudarshan Kumar<sup>c</sup>

- <sup>a</sup> Aerospace Engineering Department, Amity University, Uttar Pradesh, Noida 201301, India
- <sup>b</sup> Department of Mechanical Engineering, National Institute of Technology Surathkal, Karnataka 575014, India
- <sup>c</sup> Aerospace Engineering Department, Indian Institute of Technology Bombay, Powai, Mumbai 400076, India

#### ARTICLE INFO

Article history: Received 12 October 2009 Accepted 10 February 2011 Available online 24 February 2011

Keywords: Moore—Grietzers' model Compressor instabilities Liquid injection Rotating stall Surge

#### ABSTRACT

Two types of instabilities that occur in compression systems rotating stall and surge have an adverse effect on the compressor performance. Several techniques have been explored to minimize the effect of these instabilities. It has been observed that injection of a liquid into the compressor not only improves thermodynamic efficiencies but also results in stabilizing the system. Therefore, water and ethanol injection has been investigated as an effective tool for controlling these compressor instabilities. In the present paper a modified Moore—Greitzer model has been proposed for wet compression-based system using water and ethanol. Under this work the effect of injection of water (1) at various stages of compressor, (2) at different altitudes and (3) by varying amounts has also been presented. The effect of various parameters on wet compression such as (a) Optimum stage for liquid injection (b) Optimum amount of liquid injection and (c) Effect of altitude on liquid injection is also examined in the present work which shows that the liquid injection helps in improving the performance of compression systems in terms of increase in the stall margin and pressure rise coefficient.

© 2011 Elsevier Ltd. All rights reserved.

### 1. Introduction

Axial flow compressors are crucial components of a gas turbine engine. With increasing demand for better performance, axial flow compressors are required to operate as close as possible to the maximum efficiency line with minimum instabilities. Axial flow compressor operation is limited by two types of instabilities namely, rotating stall and surge. Greitzer [1] and Stenning [2] describe these instabilities in detail. Rotating stall involves circumferential motion of one or more stall cells. Under fully developed rotating stall, the annulus averaged mass flow remains constant. Surge on the other hand involves high amplitude, low frequency oscillation of the total annulus averaged mass flow through the compressor. Typically, the frequency associated with rotating stall is one-order magnitude higher than that of surge. Moore and Grietzer [3] describe that the two types of oscillations differ fundamentally in that the rotating stall is not axisymmetric, whereas pure surge is axisymmetric. Both these oscillations not only deteriorate the compressor performance, but also can cause substantial blade damage.

Several methods have been proposed to control the onset of these instabilities. Some of them [4–7] involve injection of

compressed air from the casing of the compressor. However this requires compressed air from a later stage of the compressor to be used for this purpose. The effect of this process on the overall compressor efficiency is as yet unclear. Inlet fogging or wet compression technique has been one of the other prominent methods used to enhance the gas turbine performance. The working fluid becomes multiphase that includes air, liquid vapor and liquid droplets subsequent to liquid injection. Wilcox and Trout [8] had proposed the use of water injection ahead of the compressor as a means of thrust augmentation. Subsequently, Hill [9] has analyzed the aerodynamic and thermodynamic effects of coolant injection on compressor performance. Recently there have been studies of water injection as a means of instability control in axial compressors. Horlock [10] reviewed the effect of water injection on compressor performance. White and Meacock [11,12] have in their studies investigated this aspect in detail. Compressor performance with water injection in the compressor inlet ducting and between stages was evaluated. Substantial improvement in the performance of the compressor was reported using water injection. Mathioudakis [13] proposed corrections in gas turbine test parameters to include the effect of water injection.

Coolant injection-based compressor has often been analyzed as a wet compression system. Zheng et al. [14] established a thermodynamic model for wet compression process. The stability of

<sup>\*</sup> Corresponding author. Tel.: +91 22 2576 7125; fax: +91 22 2572 2602. E-mail address: ampradeep@aero.iitb.ac.in (A.M. Pradeep).

Nomenclature		$p_E$ , $p_S$	static pressure at compressor outlet and compressor exit duct
Α	amplitude function of first-harmonic angular	$p_T$ , $p_l$	static pressure at throttle outlet and compressor inlet
	disturbance	U	mean rotor velocity
$A_a$	area of the compressor annulus	$V_P$	volume of plenum
$A_C$	compressor duct area	W	semi-width of compressor axisymmetric characteristic
а	reciprocal time-lag parameter of blade passage	Y	disturbance potential at compressor entrance
$a_s$	sound speed	Z	velocity ratio, $C_{xj}/C_x$
В	Greitzer parameter, $U/2a_s \times (V_p/A_aL_c)^{1/2}$	$\delta A_{\alpha}$	area of actuator through which liquid is injected into
$C_x$	axial flow velocity in compressor		the system
$C_{xj}$	velocity of liquid injection	$\varphi$	local axial flow coefficient, a function of $ heta$ and $\xi$
$F_t$	throttle characteristic function	$ec{ ilde{\phi}}_{ ilde{\phi}'}'$	velocity potential in entrance ducts
g	disturbance component in the axial direction	$ ilde{\phi}'$	disturbance velocity potential
Н	semi-height of compressor axisymmetric	$\Phi_a$	annulus averaged axial mass flow coefficient
	characteristic	$\phi_j$	local change in the flow coefficient after liquid
h	angular velocity component	-	injection
J	square of amplitude of angular disturbance of axial flow coefficient	$\Phi_j$	average change in the flow coefficient after liquid injection
$K_T$	throttle coefficient	$\eta$	axial distance measured in wheel radii
$K_G$	loss coefficient at IGV entrance	$\theta$	angular coordinate around wheel
$l_C$	non-dimensionalized total aerodynamic length $(L_c/R)$	τ	coefficient of pressure rise lag
	of compressor and ducts	$\Psi_c$	axisymmetric pressure rise coefficient
$l_I$ , $l_T$ , $l_E$	non-dimensionalized length of entrance, throttle and exit ducts	$\Psi_J$	change of pressure rise coefficient due to wet compression
m	compressor duct flow parameter	$\Psi$	total-to-static pressure rise coefficient
$m_w$	ratio of liquid injected to air mass flow rate	ξ	non-dimensionalized time parameter, Ut/R
		ρ	density

a compression system with wet compression involving water injection was evaluated by Li and Zheng [15]. The analysis involved using the Moore—Grietzer [3] model for the analysis of an axial compressor. The basic Moore—Grietzer model was modified to account for the effect of water injection. Li and Zheng [15] report substantial enhancement of the stability limits of the compressor with water injection. In the mass conservation equation as derived by Li and Zheng [15], the weighing factor of water injection term is considered as 1.0. However, the actual contribution of this term varies from 2 to 10% of what Li and Zheng [15] have considered for water injection ratios varying from 5% to 1%. This term has a significant effect on the final solution of the compression system with liquid injection.

It is a well-known fact that recovery from surge is very difficult and hence needs to be avoided [16–21]. Before the onset of surge, the compressor usually undergoes rotating stall followed by fully developed stall [16–21]. Therefore, if the initiation of rotating stall is delayed or controlled, the occurrence of surge can be completely avoided. The present study involves controlling the initiation of rotating stall using water/ethanol injection. The injection of water/ethanol also helps in reducing the compressor work and hence the turbine work output, besides augmenting thrust of the engine.

In the current study, density changes have been accounted appropriately, a factor neglected by Li and Zheng [15]. The liquid injection term in the mass conservation equation has been corrected resulting in an accurate accounting of this term to the final equations. In addition, the effect of ethanol injection on compressor performance has also been investigated. The addition of ethanol at various compressor stages is expected to enhance the stability of compressor operation and combustion efficiency because of improved mixing and better combustion with reduced pollutant emissions. The combustion limits (on mass basis) for a typical ethanol-air mixture vary from 0.0544 to 0.27 and autoignition temperature is approximately 700 K. Therefore, if the temperature

in various compressor stages is maintained below this limit, there will be no pre-ignition of the ethanol-air mixture and stability performance of the compressor will be enhanced. Both water injection as well as ethanol injection have been in the past used as methods for thrust augmentation. Hence the use of these methods would not only lead to increased thrust, but also improved stability limits. The objectives of the present study are to investigate the effect of liquid injection on the compressor performance with the following parametric variations:

- 1. At sea level and 11 km altitude.
- 2. At inlet duct and later compressor stages.
- 3. Amount of liquid injected.
- 4. Velocity of liquid injection.

# 2. Mathematical modeling of a compression system

# 2.1. Wet compression system

The basic wet compression system considered in the present analysis is shown in Fig. 1. The compression system comprises of an inlet duct, the compressor stages, outlet duct, plenum followed by a throttle. The various important dimensions used in the study are also indicated in the figure. The length of the inlet duct, the compressor and the exit duct are  $l_I$ ,  $l_C$ , and  $l_E$  respectively.

Steady state operating point of system is set by the two conditions, namely, (a) the flow through the compressor and the flow through the throttle are the same and (b) that the pressure rise through the compressor is equal to the pressure drop due to the system resistance. The following are the assumptions made in the present analysis [3]:

 Velocity and acceleration of the fluid in the plenum are negligible.

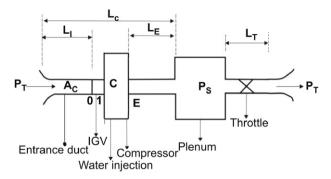


Fig. 1. Schematic diagram of a wet compression system.

- ii. Pressure in the plenum is uniform
- iii. The gas in the plenum is compressible
- iv. The compressor blades have high hub-to-tip ratio

In the present study, pressure rise coefficient, flow coefficient and disturbance amplitude are used as parameters for evaluating the performance of compressors. Pressure rise coefficient represents the work done by the compressor blades. This in turn results in the rise of pressure achieved. Flow coefficient is directly related to the mass flow rate. Stable operation of the compressor is limited by flow coefficient in terms of the surge limit and choke limit. These limits are determined by the flow coefficient. The extent of the fluctuations or disturbances is quantified using the disturbance amplitude. Increase in the disturbance amplitude is an indication of the impending instabilities.

# 2.2. Moore-Greitzer model for a dry compression system

The final equations describing the compressor operation by Moore and Greitzer [3] for a dry compression system are given below:

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\xi} = \frac{W/H}{4B^2} \left[ \frac{\Phi}{W} - \frac{1}{W} F_t^{-1}(\Psi) \right] \frac{H}{I_c} \tag{1}$$

$$\frac{d\Psi}{d\xi} = \left[ -\frac{\Psi - \Psi_{co}}{H} + 1 + \frac{3}{2} \left( \frac{\Phi}{W} - 1 \right) \left( 1 - \frac{1}{2} J \right) - \frac{1}{2} \left( \frac{\Phi}{W} - 1 \right)^{3} \right] \frac{H}{I_{c}}$$
 (2)

$$\frac{\mathrm{d}J}{\mathrm{d}\xi} = J \left[ 1 - \left( \frac{\Phi}{W} - 1 \right)^2 - \frac{1}{4}J \right] \frac{3aH}{(1 + MA)W} \tag{3}$$

The dependent variables of this system of equations are the annulus averaged pressure rise coefficient  $\psi(\xi)$ , the annulus averaged axial flow coefficient  $\Phi(\xi)$ , and the disturbance amplitude I, whose axial and circumferential partial derivatives give the local flow disturbance in the axial and circumferential directions. Independent variables include the time in wheel radians  $\xi$  and the circumferential coordinate  $\theta$ . Equation (1) is a partial differential equation in  $\xi$  and  $\theta$ , obtained from the momentum balance of the system evaluated at the compressor face; equation (2) is an ordinary differential equation in which results from averaging out the circumferential dependence in equation (1). Equation (3) is an ordinary differential equation which results from the mass balance of the plenum. The compressor characteristic  $\Psi_{\mathcal{C}}(\Phi)$  is the response of the compressor for steady axisymmetric flow. Equation (4) given below is the general Moore-Greitzer cubic form that has been used in the present analysis.

$$\Psi_c(\Phi) = \Psi_{c0} + H \left[ 1 + \frac{3}{2} \left( \frac{\Phi}{W} - 1 \right) - \frac{1}{2} \left( \frac{\Phi}{W} - 1 \right)^3 \right]$$
 (4)

The stall characteristic is defined [3] as:

$$\Psi_c(\Phi) = \Psi_{c0} + H \left[ 1 + \frac{3}{2} \left( \frac{\Phi}{W} - 1 \right) - \frac{5}{2} \left( \frac{\Phi}{W} - 1 \right)^3 \right]$$
 (5)

# 2.3. Modified Moore—Greitzer model for wet compression system

The modified Moore—Greitzer model equations for the wet compression have been described in Li and Zheng [15]. Between the upstream reservoir  $(p_T)$  and the discharge from the exit duct  $(p_s)$  is the zone where circumferential pressure variations might arise. The equation for the net pressure rise in the zone of possible angular variations can be written as:

$$\begin{split} \frac{p_s - p_T}{\rho U^2} &= \left[ NF \left( \phi_a + \phi_j \right) - \frac{1}{2} \phi_a^2 \right] - m \left( \tilde{\phi}_{\xi}' \right)_0 - \left( l_I + \frac{1}{a} + l_E \right) \left( \frac{\mathrm{d} \Phi_a}{\mathrm{d} \xi} + \frac{\mathrm{d} \Phi_j}{\mathrm{d} \xi} \right) \\ &- \frac{1}{2} (1 - K_G) h^2 - \frac{1}{2a} \left( 2 \tilde{\phi}_{\xi \eta}' + \tilde{\phi}_{\theta \eta}' \right)_0 \end{split} \tag{6}$$

Considering liquid-to-air ratio  $m_w$  then,

$$\begin{split} \frac{p_{s} - p_{T}}{\rho U^{2}} &= \left[ NF(\phi_{a}(1 + m_{w})) - \frac{1}{2}\phi_{a}^{2}(1 + m_{w})^{2} \right] - m\left(\tilde{\phi}_{\xi}'\right)_{0} \\ &- \left(l_{I} + \frac{1}{a} + l_{E}\right) \left(\frac{\mathrm{d}\Phi_{a}}{\mathrm{d}\xi}\right) (1 + m_{w}) - \frac{1}{2}(1 - K_{G}) \\ &\times (1 + m_{w})^{2}h^{2} - \frac{1}{2a}\left(2\tilde{\phi}_{\xi\eta}' + \tilde{\phi}_{\theta\eta}'\right)_{0} \end{split} \tag{7}$$

The following definitions are same as discussed in [1] and are used to represent above equation.

$$\Psi(\xi) = \frac{p_{s} - p_{T}}{\rho U^{2}}$$

$$\psi_{C}(\phi) = NF(\phi) - \frac{1}{2}\phi^{2}$$

$$l_{C} = l_{I} + \frac{1}{a} + l_{E}$$

$$\frac{1}{a} = (2NL_{R})\frac{k}{\cos^{2}\gamma}$$
(8)

The second last term of equation (7) will be neglected in the present analysis, in effect assuming that the recovery coefficient  $K_G$  at the IGV entrance is equal to 1. Finally, equation (7) can be expressed as

$$\Psi(\xi) = \psi_C \left[ (1 + m_w) \Phi_a + \left( \tilde{\phi}'_{\eta} \right)_0 \right] - l_C (1 + m_w) \left( \frac{\mathrm{d}\Phi_a}{\mathrm{d}\xi} \right) \\
- m \left( \tilde{\phi}'_{\xi} \right)_0 - \frac{1}{2a} \left( 2 \tilde{\phi}'_{\xi\eta} + \tilde{\phi}'_{\theta\eta} \right)_0 + \psi_j \tag{9}$$

A simplifying approximation of  $dh/d\theta = -g$  type is then given by Moore—Greitzer [3].Using Y a periodic function for notation for simplicity defined as  $(\tilde{\phi}')_0 = Y(\xi,\theta)$ ;  $(\tilde{\phi}'_{\xi})_0 = Y_{\theta\theta}$  also making simplifying approximation such that  $(\tilde{\phi}'_{\eta})_0 = -(\tilde{\phi}'_{\theta\theta})_0$  (9) can be written as following:

$$\begin{split} \Psi(\xi) &= \psi_{C}[(1+m_{w})\Phi_{a} - Y_{\theta\theta}] - l_{C}(1+m_{w}) \left(\frac{\mathrm{d}\Phi_{a}}{\mathrm{d}\xi}\right) - m\left(Y_{\xi}\right) \\ &- \frac{1}{2a} \left(2Y_{\xi\theta\theta} + Y_{\theta\theta\theta}\right)_{0} + \psi_{j} \end{split} \tag{10}$$

#### 2.4. Galerkin's procedure

A general transient disturbance is likely to have both angular property variations like in rotating stall and time-dependent mean flow like surge. This requires solution of the complete system of equations (1)–(3), which include derivatives that are third order in angle, but only first order in time. Therefore, Galerkin's procedure is applied as in Ref. [4] and the final Moore—Greitzer model equations are derived. The disturbance potential at the compressor inlet, Y, is represented by a single harmonic function with an unknown phase angle,  $\theta$ , as:

$$Y = WA\sin(\theta - r(\xi)) \tag{11}$$

Then substituting equation (11) in equation (10), a residual is formulated as given in the following equation (which would vanish if solution were to be exact).

$$\Psi(\xi) + l_C(1 + m_w) \frac{\mathrm{d}\Phi_a}{\mathrm{d}\xi} = \frac{1}{2\pi} \int_0^{2\pi} \left[ \psi_C((1 + m_w)\phi_a + WA\sin\zeta) + \psi_j \right] \mathrm{d}\zeta$$
(12)

The  $\sin(\theta - r)$  moment of the residual gives:

$$\left(m + \frac{1}{a}\right)\frac{\mathrm{d}A}{\mathrm{d}\xi} = \frac{1}{2W}\int_{0}^{2\pi} \sin\zeta \left[\psi_{C}((1+m_{w})\Phi_{a} + WA\sin\zeta) + \psi_{j}\right] \mathrm{d}\zeta$$
(13)

The  $\cos(\theta - r)$  moment of the residual gives:

$$-\left[\left(m+\frac{1}{a}\right)\frac{\mathrm{d}r}{\mathrm{d}\xi}-\frac{1}{2a}\right]A = \frac{1}{\pi W}\int_{0}^{2\pi}\cos\zeta\left[\psi_{C}((1+m_{W})\Phi_{a}+WA\sin\zeta)+\psi_{j}\right]\mathrm{d}\zeta$$

$$(14)$$

# 2.5. Effects of water/ethanol injection

In the present work, effect of water injection in the direction of flow is studied. When a fluid is injected into the system, it is essentially an addition of mass and momentum. Liquid injectors are arranged circumferentially and just upstream of the rotor of a compressor stage as shown in Fig. 2. In order to develop a relationship between the total pressures at two stations, additional equations are derived, as mass and momentum balance across the liquid injection. The mass balance across the liquid injector is given by:

$$\rho_a A_a (C_x + \delta C_{x1}) + \rho_i \delta A_a C_{xi} = \rho_{av} A_a (C_x + \delta C_{x2})$$
(15)

where  $C_x$  is the mean velocity in the annulus,  $C_{xj}$  is the liquid injection velocity;  $\delta C_{x1}$  and  $\delta C_{x2}$  are the velocity perturbations at

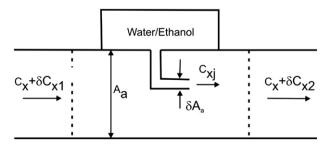


Fig. 2. Water injection into the compression system with the help of an actuator.

stations 1 and 2, respectively,  $A_{\alpha}$  is the area of the compressor annulus, and  $\delta A_{\alpha}$  is the area of opening of liquid injector,  $\rho_{\alpha}$ ,  $\rho_{j}$ ,  $\rho_{\alpha\nu}$ , and  $\rho_{\nu}$  are air, water/ethanol, air-vapor mixture and water-vapor density, respectively. It is assumed that water/ethanol droplets that sprayed are completely vaporized instantaneously.

$$m_{w} = \frac{\Phi_{j}}{\Phi_{a}}; \Phi_{j} = \frac{\rho_{j} \delta A_{a} C_{xj}}{\rho_{a} A_{a} U} = K_{j} K_{l} \frac{C_{xj}}{U}; \Phi_{a} = \frac{\rho_{a} A_{a} C_{x}}{\rho_{a} A_{a} U} = \frac{C_{xj}}{U} m_{w} = K_{j} K_{l} Z$$

After non-dimensionalizing the mass balance equation by  $\rho_{\alpha}A_{\alpha}U$  Eq. (13) reduces to:

$$\delta\phi_2 = \left(\frac{1 + m_W}{1 + m_W K_v}\right) \left[\Phi_a + \delta\phi_1 + \Phi_j\right] - \Phi_a \tag{16}$$

where  $K_l = \delta A_a/A_a$ ,  $K_w = \rho_j/\rho_a$ ,  $K_v = \rho_v/\rho_a$ ,  $z = C_{xj}/C_x$   $\delta \varphi_1, \delta \varphi_2$  are respectively flow coefficients perturbations

 $\delta \varphi_1 \delta \varphi_2$  are respectively flow coefficients perturbations upstream and downstream of the liquid injector. The momentum balance across the liquid injector is given by:

$$A_{a}(P + \delta p_{1}) + \rho_{a}A_{a}(C_{x} + \delta C_{x1})^{2} + \rho_{j}\delta A_{a}C_{xj}^{2}$$

$$= A_{a}(P + \delta p_{2}) + \rho_{av}A_{a}(C_{x} + \delta C_{x2})^{2}$$
(17)

Non-dimensionalizing the above equation by  $\rho_{\alpha}l_{\alpha}U^2$ :

$$\frac{\delta p_1}{\rho U^2} + \left( \Phi_a + 2\Phi_a \delta \phi_1 + \delta \phi_1^2 \right) + K_W \frac{\delta A_a}{A_a} (z\Phi_a)^2 
= \frac{\delta p_2}{\rho U^2} + \left( \frac{1 + m_w K_v}{1 + m_w} \right) \left( \Phi_a^2 + 2\Phi_a \delta \phi_2 + \delta \phi_2^2 \right)$$
(18)

The relationship between the static and total pressure perturbations at stations 1 and 2 are given by following equations:

$$\frac{\delta p_{t1}}{\rho U^2} = \frac{\delta p_1}{\rho U^2} + \Phi_a \delta \varphi_1 + \frac{1}{2} \delta \varphi_1^2 \tag{19}$$

$$\frac{\delta p_2}{\rho U^2} = \frac{\delta p_2}{\rho U^2} + \left(\frac{1 + m_W K_v}{1 + m_W}\right) \left(\Phi_a \delta \phi_2 + \frac{1}{2} \delta \phi_2^2\right) \tag{20}$$

then:

$$\begin{split} \frac{\delta p_{t2}}{\rho U^2} - \frac{\delta p_{t1}}{\rho U^2} &= \frac{\delta p_2}{\rho U^2} - \frac{\delta p_1}{\rho U^2} + \left(\frac{1 + m_w K_v}{1 + m_w}\right) \left(\Phi_a \delta \phi_2 + \frac{1}{2} \delta \phi_2^2\right) \\ &- \Phi_a \delta \phi_1 - \frac{1}{2} \delta \phi_1^2 \end{split} \tag{21}$$

where  $P_t$  is the total pressure in the annulus,  $\delta p_{t1}$  and  $\delta p_{t2}$  are the total pressure perturbations at stations 1 and 2 respectively.

Hence, 
$$\psi_j = \frac{\delta p_{t2}}{\rho U^2} - \frac{\delta p_{t1}}{\rho U^2}$$

#### 2.6. New compressor characteristics

It is known that liquid injection shifts the steady compressor characteristics. The equivalent compressor characteristic  $\Psi_{\rm new}$  would therefore be:

$$\Psi_{\text{new}} = \Psi_C(\Phi_a + \delta\varphi_2) + \psi_i \tag{22}$$

# 2.7. Final Moore—Greitzer equations for liquid injection

The axisymmetric characteristics need to be specified for the final Moore—Grietzer model. A cubic function as defined in Ref. [3] is used. It is obvious that if  $\Psi c$  can be expressed as sum of powers of  $\Phi$ , only even powers of  $A\sin\zeta$  in the term  $\delta\Phi_2$  will contribute to the

integral in equation (11) and only odd powers in equation (12). Inspection of these equations shows that for any form of  $\Psi c$ , the second power of A must be used. A parameter  $J(\xi)$ , is introduced in the Moore—Greitzer model to replace A:

$$J(\xi) = A^2(\xi) \tag{23}$$

$$\frac{\mathrm{d}\Psi}{\mathrm{d}\xi} = \frac{W/H}{4B^2} \left[ \frac{\Phi_a(1+m_w)}{W} - \frac{1}{W} F_t^{-1}(\Psi) \right] \frac{H}{I_C}$$

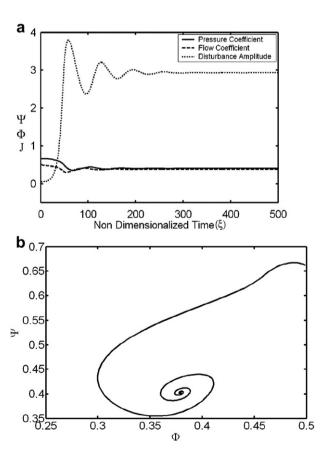
$$\Psi + l_C \frac{\mathrm{d}\Phi_a}{\mathrm{d}\xi} = \frac{1}{2\pi} \int_0^{2\pi} \left( \psi_C(\Phi_a(1+m_w) + \delta\phi_2) + \psi_j \right) \tag{24}$$

$$\Big(m+\frac{1}{a}\Big)\frac{dA}{d\xi}=\frac{1}{\pi W}\int\limits_{0}^{2\pi}\sin\zeta\Big(\psi_{C}(\varPhi_{a}(1+m_{w})+\delta\phi_{2})+\psi_{j}\Big)$$

The above set of equations (24) was solved in MATLAB using fourth order Runge–Kutta differential equation solver. A very small time step of 0.01 units was used to improve the accuracy of the computed results. This time step is an order of magnitude lower than the time step available in standard *ode45 solver* of MATLAB.

#### 3. Results and discussion

The following parameters are used to investigate the performance of a compression system under wet compression conditions [22].



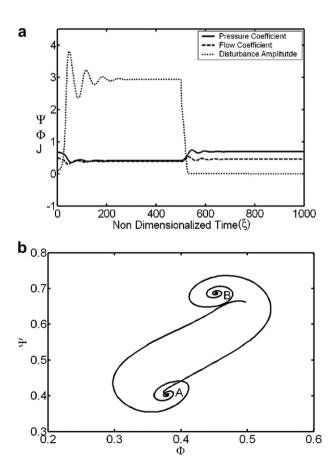
**Fig. 3.** Compressor characteristics during rotating stall (a) variation of pressure coefficient, flow coefficient, and disturbance amplitude with time (b) phase plot of flow coefficient  $(\Psi)$  and pressure coefficient  $(\Psi)$ .

$$H = 0.18$$
,  $W = 0.25$ ,  $m = 1.75$ ,  $a = 0.3$ ,  $l_C = 8$ ,  $K_T = 5.6$ 

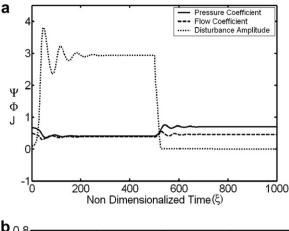
Govil and Kumar [22] reported that a stable system operation at all values of B requires a value of  $K_T \le 5.28$ . However, for a case,  $K_T \ge 5.28$ , and B = 0.5, the compression system enters into rotating stall [22]. Therefore, a point at  $K_T = 5.6$  and B = 0.5, where the system is in rotating stall has been chosen for investigating the performance of the compression system with liquid injection.

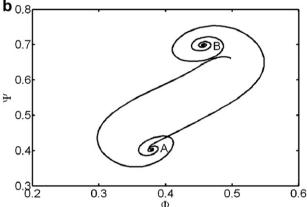
Fig. 3(a) and (b) shows the operation of a compressor system in a rotating stall mode at B = 0.5 and  $K_T = 5.6$ . The initial conditions correspond to  $I=0.05; \Psi=0.66; \Phi=0.50$ . Fig. 3(a) shows the variation of the flow coefficient, pressure coefficient, and disturbance amplitude with time. It is clear from these figures that both the flow coefficient and pressure coefficient drop and settle down at a lower off-design value as compared to normal operating conditions. It is also seen that the flow disturbance increases with time and stabilizes at a much higher value than the initial value. This indicates the occurrence of rotating stall in the compressor. The phase plot in Fig. 3(b) shows that the cycle oscillations settle to a lower pressure and flow coefficient. Since the system was operating under rotating stall, there is a drop in the final, converged values of pressure and flow coefficients. This fact was also reflected in Fig. 3(a). However these values of pressure and flow coefficients are lower than the design condition values on account of persistence of rotating stall under these conditions.

The effect of water and ethanol injection on a compression system operating at rotating stall conditions ( $K_T = 5.6$ , B = -0.5) is shown in Figs. 4 and 5 respectively. Liquid injection with a mass flow ratio of 0.06 was carried out at time t = 500. The effect of water and ethanol injection on pressure coefficient is shown in Figs. 4(a)



**Fig. 4.** Effect of water injection at t=500 (a) variation of pressure coefficient, flow coefficient, and disturbance amplitude with time (b) phase plot of flow coefficient ( $\Phi$ ) and pressure coefficient ( $\Psi$ ).





**Fig. 5.** Effect of ethanol injection at t = 500 (a) represents variation of pressure coefficient, flow coefficient, and disturbance amplitude with time (b) phase plot of flow coefficient ( $\Phi$ ) and pressure coefficient ( $\Psi$ ).

and 5(a) respectively. In both the cases, the system achieves a stable state with an increased pressure coefficient after the liquid injection. Similar behavior is observed for flow coefficient after liquid injection as shown in Figs. 5(a) and 6(a). The flow disturbances reduced to zero in both the cases after liquid injection. The operating point of the compressor system shifts from point A (unstable region) to point B (stable region) which shows that there is a significant improvement in the performance of the system. This is seen in Figs. 4(b) and 5(b), where the shift of the pressure and flow coefficients to higher values than what they were initially, can be observed. The system settles down to a state (B) which is a stable

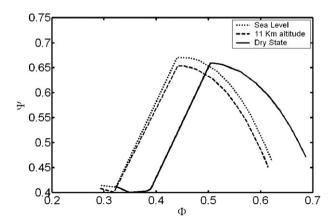


Fig. 6. Effect of 6% water injection at sea level and 11 km altitude.

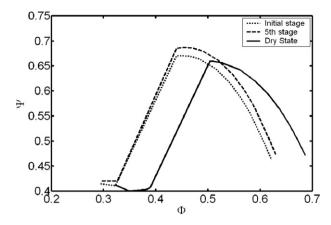


Fig. 7. Effect of 6% water injection at initial and 5th stage of the compressor.

state with improved pressure and flow coefficients. After liquid injection, the pressure coefficient (which was initially 0.66), increased to a value of  $\Psi=0.6872$  and  $\Psi=0.6992$  after water and ethanol injection, respectively. There is in fact an improvement in the pressure coefficient as compared to the design condition. Ethanol injection results in better performance improvement as compared to water injection.

# 3.1. Comparison of water and ethanol injection schemes

There are several factors that affect the operational characteristics of a compression system, namely, type of the fluid injected, amount of liquid injected, ambient conditions and the location of injection (injection either at inlet of the compressor or at later stages). Each of these parameters is of significance and needs to be evaluated carefully to qualitatively evaluate the effectiveness of liquid injection under various operating conditions.

Fig. 6 shows the compressor characteristics with and without water injection at the inlet of the compressor at sea level and 11 km altitude. The mass flow of water was 6% ( $m_W = 0.06$ ) of the air flow rate. The bold dotted line indicates the compressor characteristic for dry compression (without any liquid injection). The compressor stalls at a flow coefficient of around 0.5036. The corresponding pressure rise coefficient was 0.656. With water injection, the stalling flow coefficient has shifted to around 0.44 under sea level as well as 11 km altitude. There is also a marginal improvement in the pressure rise coefficient under sea level conditions. As compared to the dry state, the compressor with 6% water injection delays the onset of stall by shifting the stall flow coefficient to

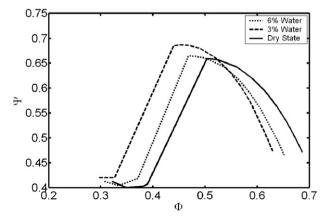
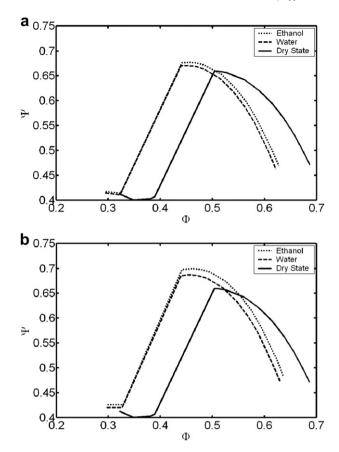


Fig. 8. Effect of varying amount water injection at sea level.



**Fig. 9.** Effect of 6% liquid injection (a) at the initial stage and (b) at the 5th stage at sea level.

substantially lower values. The differences in the compressor characteristic with different ambient conditions are due to the density of air being higher at sea level than at 11 km altitude. This leads to the ratio of density of air to water  $(1/K_w)$  at sea level to be greater than the density ratio at 11 km altitude.

The effect of water injection ( $m_w = 0.06$ ) at compressor inlet duct and the fifth stage for sea level condition is shown in Fig. 7. Injection at either the initial stage or the fifth stage leads to improved compressor performance in terms of both the pressure rise as well as stalling flow coefficient. Stalling at lower flow coefficients indicates a delay in stall/surge initiation. With water injection, a substantial delay in the onset of stall is observed. However, injection in the fifth stage leads to better performance as compared to the injection at the

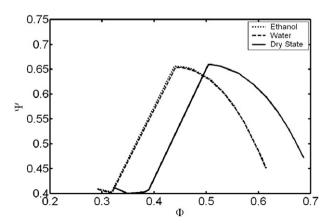


Fig. 10. Effect of 6% liquid injection at the initial stages at 11 km altitude.

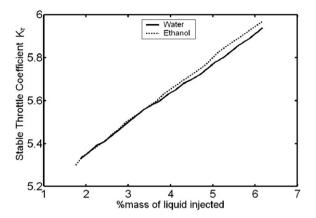


Fig. 11. Effect of amount of liquid injected into the system.

initial stage. The reason for this is again due to the higher density ratio  $(1/K_w)$  at later stages as compared to initial stages. Fig. 8 shows the effect of the amount of water injected at the fifth stage on the compressor characteristics. The dry case of the compressor characteristic is compared with the case of 3% and 6% water injection. Substantial increase in the stall limit as well as pressure coefficient can be observed. For instance, when the amount of injected water is increased from 0 to 3% and 6% (in 5th stage at sea level), the compressor stalls at  $\varphi=0.4679$  and  $\varphi=0.4551$  respectively instead of  $\varphi=0.5036$  which corresponds to the case of a dry compression. It was observed that the pressure coefficient and stall margin increased with increased amounts of water injection.

The effect of water and ethanol injection at a mass flow ratio of 0.06 is compared in Fig. 9(a). When compared to dry state compression, the injection of both the liquids leads to a substantial increase in the stall margin. There is also an improvement in pressure rise coefficient though not as high as the stall margin. Ethanol injection gives a marginally higher pressure rise coefficient as compared to water injection. The reason for this is due to the fact that ethanol has a lower density as compared to water. Therefore the air—liquid density ratio  $(1/K_w)$  for ethanol is higher than that of water. An identical improvement in both the cases is seen at initial and fifth stage as shown in Fig. 9a and b.

To achieve the same pressure rise as that of 6% water injection, 5.98% less ethanol needs to be injected than water. However, in this case, there is 2% decrease in stall margin compared to water injection. Similarly, to obtain the same stall margin as in case of water injection, 4.2% less ethanol needs to be injected and this would result in 1.1% additional pressure rise compared to water injection.

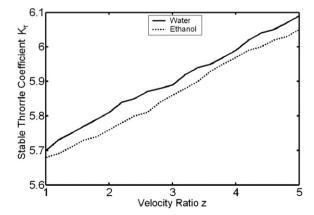


Fig. 12. Effect of velocity of injection the liquid on the stable throttle coefficient.

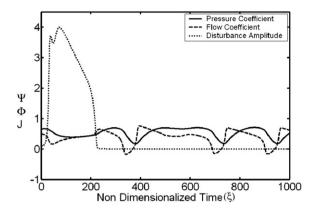


Fig. 13. Effect of water injection at 5th stage of compressor during surge.

Fig. 10 shows the improvement in the performance of the compressor at 11 km altitude. It is interesting to see that at 11 km altitude, liquid injection results in a substantial increase in the stall limit for both the cases. However, there is a marginal decrease in the pressure coefficient for liquid injection at 11 km altitude. This could be due to the reduced air—liquid  $(1/K_w)$  density ratio at 11 km altitude. Similarly, the liquid injection at fifth stage for 11 km altitude results in substantial increase in stall margin with negligible pressure rise.

Fig. 11 shows the variation in the stability limits of the compressor system with the increase in the amount of liquid injection. A stable performance can be obtained from a compression system even at higher values of throttle coefficient,  $K_T$  when the amount of liquid is increased. For instance, in the present case, when 6% water is injected at the initial stage of the system at sea level, a stable operation can be achieved at a throttle coefficient,  $K_T$  as high as 5.80. However, a change in injected fluid from water to ethanol results in a stable operation for  $K_T$  as high as 5.83. Therefore, for a given mass injected at the initial stage, ethanol injection appears to be more efficient as compared to water injection.

Fig. 12 shows the effect of velocity of liquid injection into the system for both the cases. Here, the velocity ratio indicates the ratio of liquid injection velocity to that of air flow velocity at that point. For a given velocity ratio, water injection appears to be more efficient than ethanol injection. This is due to the higher density of water which essentially leads to an increased momentum of the injected fluid and helps in stabilizing the compressor operation at higher throttle coefficients when compared with ethanol injection.

In order to study the effect of liquid injection on surge of the axial compressor, water and ethanol injection was used at a higher  $K_T$  value of 6.6 which corresponds to surge. Fig. 13 shows the response of the compression system with 6% water injection. The disturbance amplitude that was initially high reduced to zero with water injection. But the pressure and flow coefficient fluctuations are not controlled with water injection. Similar results were obtained with ethanol injection as well. This means that liquid injection has negligible effect on surge. However, since it has already been observed that liquid injection does control rotating stall, use of liquid injection during rotation stall would avoid the compressor going to surge condition.

#### 4. Conclusions

The behavior of an axial compressor subjected to wet compression is investigated in this paper and it is found that liquid injection into the unstable system stabilizes the system operation and also improves the thermodynamic performance of compression system. The effect of water injection has been compared with ethanol

injection into an unstable compression system. It is found that the gains in terms of stall margin improvement and pressure rise coefficient are much better for ethanol injection case as compared to the case of water injection. This is due to the fact that the density of ethanol is less than that of water. Therefore, it appears to be more beneficial for the airline industries to employ the ethanol-based liquid injection scheme as compared to water injection because the combustion of ethanol vapor would also release additional energy during the combustion, this helping in further thrust augmentation. During the current study, it was observed that injection of a liquid at later stages results in improved stall margin and higher pressure rise as compared to liquid injection in the initial stages. It was also observed that ethanol injection is a better option than water for improving the performance of the compression system.

# Acknowledgements

The authors would like to thank IRCC (Industrial Research and Consultancy Centre), Indian Institute of Technology Bombay for funding this work through the Summer Internship Program. We would like to thank Prof. S. K. Sane (Ex-Professor, Aerospace Engineering Department, IIT Bombay) for the fruitful discussions which we had with him during the course of this work.

#### References

- E.M. Greitzer, The stability of pumping systems the 1980 Freeman scholar lecture, Transactions of the ASME Journal of Fluid Engineering 103 (1981) 193–242
- [2] A.H. Stenning, Rotating stall and surge, Transactions of the ASME Journal of Fluid Engineering 102 (1980) 14–20.
- [3] F.K. Moore, E.M. Greitzer, A theory of post-stall transients in axial compression systems, part I: development of equations, The ASME Journal of Engineering for Gas Turbines and Power 108 (1986) 68–76.
- [4] I.J. Day, T. Breuer, J. Escuret, M. Cherrett, A. Wilson, Stall inception and the prospects for active control in four high speed compressors, ASME Transactions Journal of Turbomachinery 121 (1999) 18–27.
- [5] K.L. Suder, M.D. Hathway, S.A. Thorp, A.J. Strazisar, M.B. Bright, Compressor stability enhancement using discrete tip injection, ASME Transactions Journal of Turbomachinery 123 (2001) 14–23.
- [6] C. Nie, G. Xu, X. Cheng, J. Chen, Micro air injection and its unsteady response in a low-speed axial compressor, ASME Transactions Journal of Turbomachinery 124 (2002) 572–579.
- [7] J.F.E. Williams, M.F.L. Harper, D.J. Allwright, Active stabilization of compressor instability and surge in a working engine, ASME Transactions Journal of Turbomachinery 115 (1993) 68–75.
- [8] E.C. Wilcox, A.M. Trout, Analysis of Thrust Augmentation of Turbojet Engines by Water Injection at Compressor Inlet Including Charts for Calculating Compression Processes with Water Injection. NACA, 1950, Technical report no. 1006.
- [9] P.G. Hill, Aerodynamic and thermodynamic effects of coolant injection on axial compressors, Aeronautical Quarterly 12 (1963) 331–348.
- [10] J.H. Horlock, Compressor Performance with Water Injection (2001) ASME Paper 2001-GT-0343.
- [11] A.J. White, A.J. Meacock, An Evaluation of the Effects of Water Injection on Compressor performance (2003) ASME Paper GT-2003-38237.
- [12] A.J. White, A.J. Meacock, The effect of water injection on multispool gas turbine behavior, The ASME Journal of Engineering for Gas Turbines and Power 128 (2006) 97–102.
- [13] K. Mathioudakis, Analysis of the effects of water injection on the performance of a gas turbine, The ASME Journal of Engineering for Gas Turbines and Power 124 (2002) 489–495.
- [14] Q. Zheng, Y. Sun, S. Li, Y. Wang, Thermodynamic analysis of wet compression process in the compressor of gas turbine, ASME Transactions Journal of Turbomachinery 125 (2003) 489–496.
- [15] M. Li, Q. Zheng, Wet Compression System Stability Analysis (2004) ASME Paper No. GT2004-54018.
- [16] H.W. Emmons, R.E. Kronauer, J.A. Rockett, A survey of stall propagationexperiment and theory, Journal of Basic Engineering 81 (1959) 409–416.
- [17] E.M. Greitzer, Surge and rotating stall in axial flow compressors, part I: theoretical compression system model, Journal of Engineering for Gas Turbines and Power 98 (1976) 190–198.
- [18] E.M. Greitzer, Surge and rotating stall in axial flow compressors, part II: experimental results and comparison with theory, Journal of Engineering for Gas Turbines and Power 98 (1976) 199–217.

- [19] F.K. Moore, A theory of rotating stall of multistage compressors: part I Small disturbances, Journal of Engineering for Gas Turbines and Power 106 (1984) 313–321.
- [20] F.K. Moore, A theory of rotating stall of multistage compressors: part II Finite disturbances, Journal of Engineering for Gas Turbines and Power 106 (1984) 321–326.
- [21] I.J. Day, E.M. Greitzer, N.A. Cumpsty, Prediction of compressor performance in rotating stall, Journal of Engineering for Gas Turbines and Power 100 (1978) 1–14.
- [22] K. Govil, S. Kumar, Analysis of non recoverable stall and other instabilities using Moore Greitzer model, in: 44th AlAA/ASME/SAE/ASEE Joint Propulsion conference and exhibit, Hartford, USA, AlAA 2008-4583-377, 2008.