Selective Image Smoothing and Feature Enhancement Using Modified Shock Filters

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***Abstract*—Shock filters are widely used for image enhancement and deblurring. These filters make use of nonlinear hyperbolic Partial Differential Equations (PDEs) in order to sharpen the edges. However, in many practical cases images are corrupted by noise and other kind of degradations. Convensional shock filters are not suitable in such cases as they enhance the noise present in the image. Hence, the idea of combining shock filters with diffusion yield good results. In this paper we propose a modified "diffusion coupled shock filter." The proposed method makes use of an 'adaptive diffusion term' which limits the extent of smoothing on important edges making them more sharper. The experimental results demonstrate the efficiency of the proposed method to control diffusion and to make the reconstruction more reliable.**

***Index Terms*—Shock filter; Diffusion; Deblurring; Image Enhancement.**

I. Introduction

Partial Differential Equations (PDEs) and variational methods play an important role in image enhancement and denoising [1], [2], [3], [4]. These models allow the generation of an image scale space where-in the image is simplified while preserving or enhancing important features like edges, lines, corners etc. One of the notable achievements in the field of image enhancement was the introduction of shock filters by Osher and Rudin [5]. This filter belongs to the class of hyperbolic PDEs which create strong discontinuities at the image edges. A major drawback of this method is that it will enhance the noise present in the image as well. Many modifications were suggested by researchers to address this issue. In [6] and [7], the authors apply a smoothing kernel before applying the shock filter, so that the noise get smoothed out and the shock filter enhances the edges with less enhancement of noise. This method was not much encouraged because the noise is smoothed out at the cost of edges. Alvarez and Mazorra(A-M) [1] combine shock filters and anisotropic diffusion and add a smoothing kernel for the estimation of the direction of the edges. Similar modifications were suggested in [8] and [9] also. All these filters use a variation of Mean Curvature Motion(MCM) [10] as the diffusion term. In MCM, the level curves of the image evolve with a speed proportional to their mean curvature value. The continued application of MCM causes blurring and deformation of edges (usually the edges appear more curvy) and will result in a constant intensity image, if the evolution is not stopped after a finite number of iterations. In this paper, we propose a selective smoothing-enhancing filter which enhances the image without fading out the edges, corners and other finer information present in the image. We use a modified diffusion term along with the shock term in order to reduce diffusion at the edges. This will preserve the location and sharpness of edges while smoothing homogeneous areas. The shock term present in the filter will give a sharp shock to the important edges. The experimental results are provided and compared qualitatively and quantitatively with the most relevant image enhancing methods in the literature.

This paper is organized as follows. Section 2 describes the mathematical background of the shock filters and cover some of the relevant shock filters in the literature. Section 3 explains the proposed method, its mathematical formulation and the advantages of using them for image enhancement. Section 4 explains the numerical schemes employed for solving the proposed PDE. Section 5 highlights the experimental results and their comparison with the existing methods for image enhancement. The last section concludes the work.

II. Background of Shock Filters

The 'classical' shock filter formulated in [5] is:

$$\frac{\partial I}{\partial t} = -sign(I_{\eta\eta})|\nabla I| \tag{1}$$

where η is in the direction of the gradient ∇I of image function I , $|.|$ denotes the Euclidean norm and $sign$ function is defined as:

$$sign(x) = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ +1 & \text{if } x > 0 \end{cases}$$

Note that the PDE in (1) is a hyperbolic PDE, with the Neumann Boundary condition:

$$\frac{\partial I}{\partial \vec{n}} = 0 \tag{2}$$

where $\vec{n}$ is the outward normal to the level curve, and

$$I(x, y, 0) = I_0(x, y) \tag{3}$$

is the initial image. Throughout this paper we assume the boundary condition (2) and the initial condition (3) for all the

PDEs, unless stated otherwise. The hyperbolic PDE in (1) is an anisotropic one, which will give a "shock" in the direction of gradient and will not have any effect on the direction of isophotes. This will make the edges more prominent, but will enhance the noise as well. This property makes them a bad choice for image enhancement because in most practical scenarios the images are corrupted by noise.

A. Denoising Shock Filters

An improvement of classical shock filter used in the papers [6] and [7] is :

$$\frac{\partial I}{\partial t} = -sign(\nabla^2(G_\sigma * I))|\nabla I| \tag{4}$$

where G_σ is a Gaussian kernel with standard deviation σ, '*' denotes the linear convolution operation and the other symbols are as in (1). The image is treated with a Gaussian kernel to eliminate the effect of noise and the shock filter hardly amplifies the noise components in the image. However, the noise smoothing property of this filter is tightly coupled with the standard deviation parameter σ of the Gaussian kernel. When σ is large, the image edges are badly affected by the smoothing process whereas, if the parameter is small, the noise will not be removed effectively.

B. Shock Coupled Diffusion

In [1], the MCM based diffusion is combined with a shock term to avoid the noise enhancing property of the classical shock filter. It has been formulated as:

$$\frac{\partial I}{\partial t} = -sign(G_\sigma * I_{\eta\eta})|\nabla I| + \lambda I_{\xi\xi} \tag{5}$$

here λ denotes the control parameter which is a positive scalar value, η is the direction along the gradient and ξ is the direction perpendicular to the gradient. This anisotropic filter will act like a "shock" filter in the direction of gradient and a diffusion filter in the direction perpendicular to the gradient. The term $I_{\xi\xi}$ represents MCM, where each of the level curves in the image move in the normal direction with a speed proportional to their mean curvature value. The parameter λ controls the amount of diffusion. This filter will denoise the image quite well while enhancing edges. However, the continued application of MCM causes the curvy edges to become more curvy. So, as the number of iterations increase, the resulting image will loose the exact shape and sharpness of the edges.

C. Complex Diffusion with Shock

The use of complex diffusion for noise smoothing and edge preservation was first introduced in the area of vision computing by Sochen et al. in [11]. The complex diffusion works under the assumption that the image can be considered like a complex function having both real and imaginary components. The shock filter proposed in [11] is:

$$\frac{\partial I}{\partial t} = -\frac{2}{\pi}\, arctan(a\ I_{\eta\eta}) + \lambda I_{\xi\xi} \tag{6}$$

where the parameter a controls the sharpness of the slopes at zero, $arctan$ is a "soft" *sign* function. This modification will sharpen the regions with large gradient magnitude (especially at the zero crossings in the Laplacian) faster, compared to smoother areas.

It is quite logical to incorporate a time dependency in diffusion process. The idea of explicitly incorporating the time t in Perona-Malik diffusion scheme is elaborated in [3]. The same approach can be utilized in case of shock filters as well. Since the noise component will be dominant during the earlier stages of evolution process, the effect of shock should be minimum in order to avoid the noise getting enhanced. The filter given in (6) is modified as:

$$\frac{\partial I}{\partial t} = -\frac{2}{\pi} arctan(a\ I_{\eta\eta}\ t)|\nabla I| + \lambda I_{\xi\xi} \tag{7}$$

Further it is shown in [11] that, in a complex diffusion process with the diffusion coefficient $c = re^{i\theta}$, as $\theta \to 0$ the imaginary part of image can be considered as a smooth second derivative of the initial signal factored by θ and time t. That is,

$$\lim_{\theta \to 0}\left(\frac{Im(I)}{\theta}\right) = t\ \Delta G_\sigma * I_0 \tag{8}$$

where $Im(.)$ is the imaginary part of Image I, Δ is the Laplacian operator. The standard deviation σ for the Gaussian kernel is calculated as $\sigma = \sqrt{2t}$. Therefore, the complex shock filter by Sochen, Gilboa and Zeevi (SGZ) in [8] is

$$\frac{\partial I}{\partial t} = -\frac{2}{\pi} arctan(a\ Im(\frac{I}{\theta}))|\nabla I| + \lambda I_{\xi\xi} \tag{9}$$

where λ is the diffusion controlling parameter which is a positive scalar value.

III. Proposed Method

Though the filters in (5) and (9) denoise the image while enhancing the edges, they fail to take the full advantage of shock filtering since there is no edge stopping function in the diffusion term. Therefore, the diffusion is not stopped at the edges resulting in less sharp features as the iteration continues and the edges get blurred and more curvy after a finite number of iterations. The diffusion term $I_{\xi\xi}$ in both (5) and (9) is MCM (see [10] for details):

$$I_{\xi\xi} = |\nabla I| div\left(\frac{\nabla I}{|\nabla I|}\right) \tag{10}$$

where div denotes the divergence operator and the other symbols are as defined previously. Since each of the level sets in the image try to evolve continuously and shrink to a point in MCM, it is natural to incorporate an "edge stopping" function to the diffusion term to stop the evolving level-sets when they reach important image edges. Therefore, we propose to use an image selective smoothing term instead of the pure MCM as in (5) and (9). The shock term along with this nonlinear diffusion term will enhance the image, penalizing less on edge features. The shock filter in (5) can be reformulated with proposed modification as:

$$\frac{\partial I}{\partial t} = -sign(G_\sigma * I_{\eta\eta})|\nabla I| + c\ g(|G_\sigma * \nabla I|)I_{\xi\xi} \tag{11}$$

where $I_{\xi\xi} = |\nabla I| \; div\left(\frac{\nabla I}{|\nabla I|}\right)$ represents a degenerate diffusion term which diffuses in the direction orthogonal to the gradient ∇I and does not diffuse at all in the direction of the gradient. The degenerate diffusion term will assure the smoothing of the image on either sides of the edges with minimal smoothing on the edges itself. $G_\sigma * \nabla I$ is the Gaussian convolved version of the gradient image ∇I. The function $g(|G_\sigma * \nabla I|)$ is chosen as a decreasing function of the image gradient which tends to zero as $|G_\sigma * \nabla I| \to \infty$, in order to prevent smoothing at edges.

$$g(|G_\sigma * \nabla I|) = \frac{1}{1 + \frac{(|G\sigma * \nabla I|)^2}{K^2}}. \tag{12}$$

If the gradient magnitude $|G_\sigma * \nabla I|$ has a small value in the neighborhood of a point x, then x is considered as an interior point and the diffusion is strong in that area. On the other hand, if $|G_\sigma * \nabla I|$ has a very large value in the neighborhood of x, then that point corresponds to an edge and the diffusion is stopped at that point. Thus, this selective smoothing method will preserve the location and sharpness of the edges even after a large number of iterations. The standard deviation parameter σ in G_σ decides the extent of edge details to be kept while processing the image. Incorporating $g(|G_\sigma * \nabla I|)$ to the diffusion term in (5) will inhibit further shrinkage of level lines when it reaches important boundaries while the shock term makes the edges sharper. The parameter K is a contrast parameter as in [3].

Similarly, we can modify the complex shock filter equation in (9). The filter in (9) can be reformulated with proposed modification as:

$$\frac{\partial I}{\partial t} = -\frac{2}{\pi} arctan\left(a \; Im\left(\frac{I}{\theta}\right)\right) |\nabla I| + \lambda g(|G_\sigma * \nabla I|) I_{\xi\xi} \tag{13}$$

where $g(|G_\sigma * \nabla I|)$ is as in (12) and λ is the parameter that controls the extent of smoothing. The proposed filters in (11) and (13) will not introduce any local extrema points and the total variation is not increasing, hence over and under-shoots will never occur in these filters.

IV. Numerical Implementation

Since the shock filters are hyperbolic PDEs, the central difference scheme cannot be applied for the implementation of shock term in (11) and (13). So we use the *upwind* scheme proposed in [12] for the shock term. The spatial step h is assumed to be 1 and Δt is the time step. The *upwind* scheme for evaluating $|\nabla I|$ in the shock term in (11) and (13) is

$$|\nabla I| = \sqrt{D_x^2 + D_y^2} \tag{14}$$

where

$$\begin{aligned} D_x &= minmod(I_x^+(x,y), I_x^-(x,y)) \\ D_y &= minmod(I_y^+(x,y), I_y^-(x,y)), \end{aligned} \tag{15}$$

the $minmod$ operator is defined as:

$$minmod(x,y) = \begin{cases} min(|x|,|y|) & \text{if } xy > 0 \\ 0 & \text{Otherwise} \end{cases}$$

and

$$\begin{aligned} I_x^+(x,y) &= I(x+1,y) - I(x,y) \\ I_x^-(x,y) &= I(x,y) - I(x-1,y) \\ I_x(x,y) &= (I_x^+ + I_x^-)/2 \\ I_y^+(x,y) &= I(x,y+1) - I(x,y) \\ I_y^-(x,y) &= I(x,y) - I(x,y-1) \\ I_y(x,y) &= (I_y^+ + I_y^-)/2 \end{aligned} \tag{16}$$

The central difference scheme is used for the diffusion term in (11) and (13).

$$I_{\xi\xi} = \frac{I_{xx}|I_y|^2 - 2I_{xy}I_xI_y + I_{yy}|I_x|^2}{1 + |I_x|^2 + |I_y|^2} \tag{17}$$

where

$$\begin{aligned} I_{xx} &= I(x+1,y) - 2I(x,y) + I(x-1,y) \\ I_{yy} &= I(x,y+1) - 2I(x,y) + I(x,y-1) \\ I_{xy} &= (I(x+1,y+1) + I(x-1,y-1) \\ &\quad -I(x-1,y+1) - I(x+1,y-1))/4 \end{aligned} \tag{18}$$

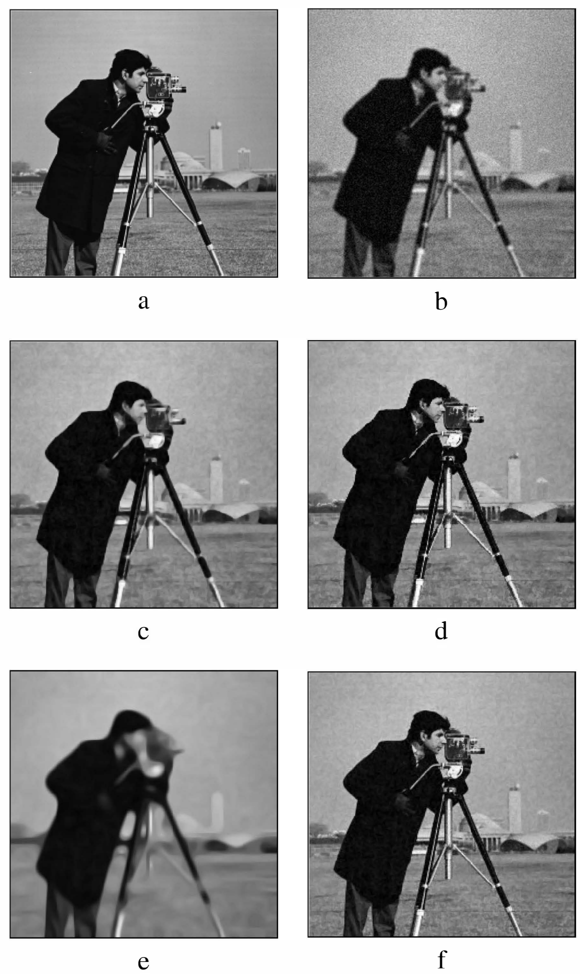

Fig. 1. (a) Original image, (b) Blurred and noisy image, (c)-(d) results of A-M filter and proposed filter in (11) after 100 iterations, (e)-(f) results of SGZ filter and proposed filter in (13) after 100 iterations, respectively.

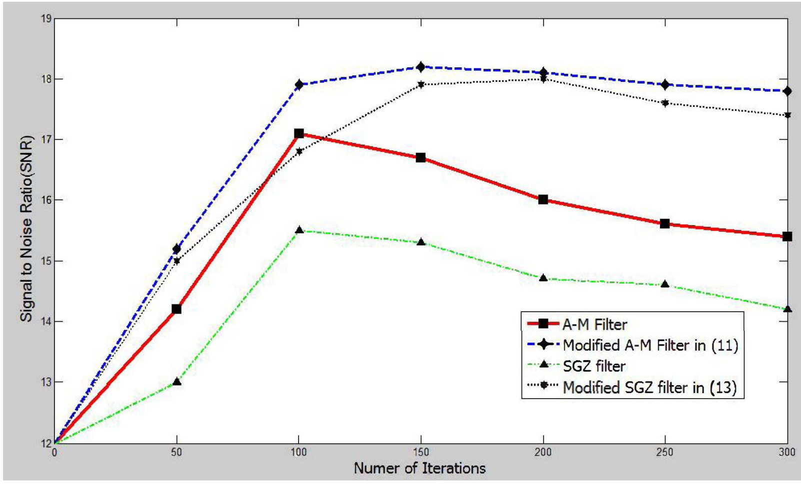

Fig. 2. The SNR values of different methods in various iterations for the image 'cameraman'

V. Experimental Results

We use the standard test image "cameraman" to test the performance of our algorithm. The original image is corrupted by Gaussian noise making the SNR (Signal to Noise Ratio) of the noisy image 12 dB. We have chosen a value of 15 for the parameter K, a value of 10 for the standard deviation σ in the Gaussian smoothing function G in the expression (12). We have generated an out-of focus blur for the image using a Gaussian smoothing function with standard deviation $\sigma = 17$. The time step is chosen as $\Delta t = 0.1$. For complex diffusion the following parameters are set; angle $\theta = \frac{\pi}{1000}$, diffusion controlling parameter $\lambda = 0.5$, the slope controlling parameter $a = 0.3$.

Fig 1(a) show the original image used for experiments and Fig 1(b) shows the blurred and noisy image. The results of applying the A-M and SGZ shock filters in equations (5), (9) respectively and the results of the proposed filters (11) and (13) after 100 iterations are shown in Fig 1. One can see from Fig 1 that the proposed method is very effective in preserving and sharpening the image features when compared to the methods already available in the literature.

The graph in Fig 2 shows the SNR values in each iteration for the image "cameraman." It can be observed from Fig 2 that the SNR values of the results for the A-M scheme and SGZ scheme are deteriorating much faster than the proposed schemes after a finite number of iterations. In the proposed schemes, the reduction of the SNR is mainly due to the shock term present in the filter, whereas in A-M and SGZ the SNR decrement is not only due to the shock term, but also due to the uncontrolled diffusion across the edges. The proposed method will preserve the edges in the image even after a large number of iterations while denoising the image. The methods based on (5) and (9) will diffuse the important image features such as edges as the iterations are continued resulting in less sharp features.

VI. Conclusion

An image selective smoothing and enhancement method is proposed with a modification in the diffusion term of the shock filters in [1], [8]. This proposed modification will stop the evolution of isophotes in the regions dominated by edges and the shock component, present in the filter will create sharp discontinuities at the inflection points resulting in the enhancement of edges. This method is implemented and tested for various images and the results provided in Section 5 demonstrate the efficiency of the proposed method.

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