# Significance of Reactive Power Loss and Its Application to System Voltage Stability

Tukaram Moger*, Teena Johnson* and Thukaram Dhadbanjan**

***Abstract*—In this paper, the significance of reactive power loss and its application to system voltage stability is presented. The reactive power loss allocation at the load-buses is computed from the reactive power support and loss allocation algorithm using modified Y-bus approach. Further, it is computed for the various load conditions in the system upto the maximum loadability point. The proposed approach is illustrated on a sample 5-bus system and also tested on a 11-bus practical equivalent system of Indian southern region power grid. A comparative analysis is also carried out with the continuation power flow method to highlight the features of the proposed approach. It can be observed from the simulation results that the reactive power loss allocation at load-buses give the clear indication about the system reactive power issues, which in-turn give an indication about the system voltage instability/collapse problem.**

***Index Terms*—reactive power loss, voltage stability, modified Y-bus method, weak buses.**

## I. Introduction

Modern electric power utilities are facing many challenges due to ever-increasing complexity in their operation and structure. In the recent past, one of the problems that got wide attention is the power system instability [1]–[3]. With the lack of new generation, transmission facilities and over exploitation of the existing facilities geared by increase in load demand make these types of problems more imminent in modern power systems.

Power transfer in the transmission network causes power losses due to the resistances and reactances of the network. The amount of reactive power losses in the line is significant. But, the reactive power loss is also dependent on the active power flows in the system. Moreover, a unity power factor load can cause a flow of reactive power in the network [4], [5]. Especially, under heavily loaded conditions, the amount of reactive power loss may exceed the total reactive power demand of the system. Hence, reactive power loss should be considered in the evaluation of systems total reactive power requirement. Developing a fair and adequate method of determining the reactive power loss allocation may give more realistic economic signals to market participants and system operator regarding the system reactive power issues. This would offer system operator with better tools to strengthen the system security.

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For proper allocation of power, tracing the path of power flow from generator bus to load-bus is necessary. This path can be identified using power-flow tracing methods. Based on certain assumptions and approximations, several power tracing methodologies have been reported in the literature [6]–[13].

Among the circuit based allocation approaches [14]–[20], the Z-bus matrix and modified Y-bus matrix methods are very efficient and suitable for use in real system as they integrate the network characteristics in terms of network equations directly. Since it is based on a power flow solution, it can consider the system non-linearity accurately.

It is known fact that reactive power is directly linked to voltage stability [4], [5]. Inadequate reactive power has led to voltage collapses and has been a major cause of several recent major power outages worldwide [1]–[3]. Moger and Dhadbanjan proposed a new approach for identification of critical load-buses based on reactive power loss [21]. This identification is carried out under a normal loading condition without loading the system upto maximum loadability point unlike in the case of continuation power flow method (CPF) [22]. However, this paper mainly focuses on the reactive power loss allocation at load-buses for the various load conditions upto the system maximum loadability point and its significance on system voltage stability.

This paper presents the impact of reactive power loss allocation at load-buses on voltage stability analysis. Modified Y-bus approach is used to determine the reactive power loss allocated at load-buses with the reactive power support data. Further, it is computed for the various load conditions in the system upto the maximum loadability point. The reactive power produced by the line charging capacitances is accounted by integrating the shunt parts of the transmission line into the nearby buses. The proposed approach is illustrated on sample 5-bus system and also tested on 11-bus practical equivalent system of Indian southern region power grid. Later, a comparative analysis is carried out with the continuation power flow method [22] to highlight the features of the proposed approach.

## II. Approach for reactive power loss allocation

### A. Transmission line modelling

The line charging capacitances can be considered as sources of providing reactive power to the system. The equivalent model of transmission line is shown in Fig. 1. The reactive powers ($Q_{c,m}$ and $Q_{c,n}$) produced by the line shunt admittances ($Y_{sh}/2$) are transferred into the nearby buses with an assumption that the voltages of the shunt admittances are same as that of the nearby buses. The bus voltages can be obtained from the power flow results.

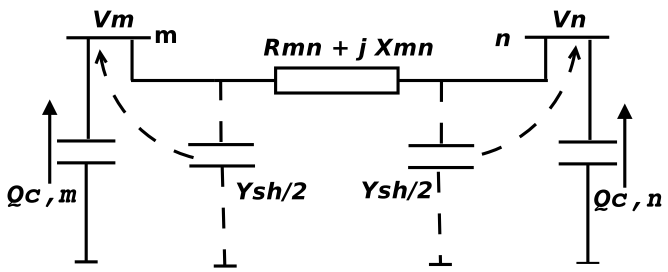


Fig. 1. Transmission line equivalent model

$$Q_{c,m} = \Im(V_m^2 Y_{sh}/2) \quad (1)$$

$$Q_{c,n} = \Im(V_n^2 Y_{sh}/2) \quad (2)$$

The total reactive power produced at all generator and load-buses are calculated considering all the reactive power sources or sinks including line charging capacitances at the respective buses.

### B. Reactive power loss allocation to load-buses

The system comprises of $n$ buses with $ng$ generator buses and $nl$ load-buses. For the given system, the performance equation of the network under steady state condition is given by,

$$[I_g] = [Y_{gg}][V_g] + [Y_{gl}][V_l] \quad (3)$$

$$[I_l] = [Y_{lg}][V_g] + [Y_{ll}][V_l] \quad (4)$$

where $[I_g]$ = $[I_1, \ldots, I_{ng}]^T$ is the injected currents of generator buses; $[I_l]$ = $[I_{ng+1}, \ldots, I_n]^T$ is the injected currents of load-buses; $[V_g]$ = $[V_1, \ldots, V_{ng}]^T$ is the complex generator bus voltages; $[V_l]$ = $[V_{ng+1}, \ldots, V_n]^T$ is the complex load-bus voltages; and $[Y_{gg}]$, $[Y_{gl}]$, $[Y_{lg}]$, $[Y_{ll}]$ are the corresponding partitioned matrices of the bus admittance matrix. Equation (3) can be rewritten in terms of load-bus currents and generator bus voltages as

$$[I_g] = [K_{gl}][I_l] + [Y''_{gg}][V_g] \quad (5)$$

where $[K_{gl}]$ = $[Y_{gl}][Z_{ll}]$, $[Y''_{gg}]$ =$[Y_{gg}]$-$[Y_{gl}][Z_{ll}][Y_{lg}]$ and $[Z_{ll}]$ = $[Y_{ll}]^{-1}$

The main objective of the proposed work is to get the generators' contributions to meet load demand and losses in the system. To achieve this objective, using the circuit theory, the generator bus voltage ($V_g$) in (5) is being replaced as a function of load-buses voltage, i.e., $V_g = f(V_l)$. Superposition theorem can be applied to deduce $V_g$ in terms of $V_l$. However, all generators' current injections have to be replaced by its equivalent admittances in the circuit. Using the load flow results, the equivalent shunt admittance $Y_{g_j}$ at a generator node $j$ can be obtained as follows

$$Y_{g_j} = \frac{1}{V_{g_j}} \left( \frac{-S_{g_j}}{V_{g_j}} \right)^* \quad (6)$$

where (*) means conjugate, $S_{g_j}$ is the generator apparent power at node $j$ and $V_{g_j}$ is the generator voltage at node $j$.

Now, these equivalences are added to update the corresponding diagonal elements of Y-bus matrix. Then from (3), the generator bus voltage $V_g$ can be solved as

$$[V_g] = -[Y'_{gg}]^{-1}[Y_{gl}][V_l] \quad (7)$$

where $[Y'_{gg}]$ is the modified matrix of $[Y_{gg}]$. In (7), assuming

$$[Y_b] = -[Y'_{gg}]^{-1}[Y_{gl}] \quad (8)$$

Then, (7) can be written as

$$[V_g] = [Y_b][V_l] \quad (9)$$

The voltage contribution of each load-buses to the generator bus is given by,

$$V_{g_j} = \sum_{i=1}^{nl} Y_{b j,i} * V_{l_i} \quad (10)$$

It can be observed from (10) that voltage at generator bus $j$ is the sum of contribution of voltages from all load-buses.
By substituting (9) into (5), the generator current can be obtained as,

$$I_g = [K_{gl}][I_l] + [Y_c][V_l] \quad (11)$$

where $[Y_c] = [Y_{gg}][Y_b]$

In order to calculate the generators share/contribution to meet the load demand and losses, the vectors $[I_l]$ and $[V_l]$ need to be considered as a diagonal matrix. Taking the conjugate of (11) and pre-multiplying it by $[V_g]$, the complex power of generators is given by

$$\begin{aligned}[V_g]_{ng\times ng}[I_g^*]_{ng\times nl} &= [S_{gen-contrb}]_{ng\times nl} \\ &= [V_g]_{ng\times ng}[K_{gl}^*]_{ng\times nl}[I_l^*]_{nl\times nl} \\ &\quad + [V_g]_{ng\times ng}[Y_c^*]_{ng\times nl}[V_l^*]_{nl\times nl} \end{aligned} \quad (12)$$

The reactive power contribution of all generators is obtained by,

$$[Q_{gen-contrb}]_{ng\times nl} = Im \ ([S_{gen-contrb}]_{ng\times nl}) \quad (13)$$

A simplified version of (13), the reactive power contribution of generator $j$ to load-bus $i$ is given by:

$$Q_{gen-contrb_j} = \sum_{i=1}^{nl} Q_{gen-contrb_{j,i}} \quad (14)$$

Using (14), the reactive power loss allocation at each load-bus $i$ is given by,

$$Q - loss_i = \sum_{j=1}^{ng} Q_{gen-contrb_{j,i}} - Q_{l_i} \quad (15)$$

where $Q_{l_i}$ : net reactive power demand at load-bus $i$.

## III. Case studies

### A. Sample 5-bus system: An illustrative example

A sample 5-bus system (See Fig. 2) is considered to illustrate the significance of Q-loss and its application to system voltage stability analysis. System has 2 generators and 3 loads. The lines L1 (connecting buses 1 and 3), L2, L3 and L4 are of lengths 50, 150, 100 and 100 kms respectively. The 400 kV line parameters per 100 km are $R$ = 0.002 p.u., $X$ = 0.020 p.u. and $b/2$ = 0.25 p.u. The initial base-case load is 1100 MW and 555 MVAr.

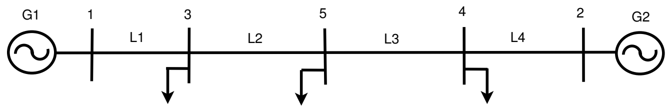


Fig. 2. Single-line diagram of 5-bus system

For any operating point, the power-flow analysis on the system is performed. Then, the net Q-support at each generator and load-buses (including the other reactive sources/sinks and line charging capacitances at the respective buses) is calculated. For illustration, the result of the system under peak-load condition is presented. The peak-load is 1265 MW and 638.25 MVAr, which is 15% more than the base-case loading condition. The power-flow results of the system under peak load condition is presented in Table I along with the computation of net Q-support at each generator and load-buses including the line susceptances and other reactive sources or sinks at their respective buses. The system has active and reactive power losses of 20.296 MW and 202.964 MVAr respectively. The Table II shows Q-loss allocations at load-buses obtained using this new approach for system under peak-load condition.

TABLE I
LOAD FLOW RESULT FOR SAMPLE 5-BUS SYSTEM

| Bus | Voltage | | Generation | | Load | | Net |
|---|---|---|---|---|---|---|---|
| No. | Mag. (p.u.) | Angle (deg.) | PG (MW) | QG (MVAr) | PD (MW) | QD (MVAr) | MVAr |
| 1 | 1.00 | 0 | 710.2964 | 366.954 | - | - | 379.454 |
| 2 | 1.00 | -0.14343 | 575 | 296.5375 | - | - | 321.5375 |
| 3 | 0.95732 | -4.02737 | - | - | 402.5 | 207 | -161.1773 |
| 4 | 0.93055 | -6.84352 | - | - | 345 | 172.5 | -129.204 |
| 5 | 0.90424 | -9.71504 | - | - | 517.5 | 258.75 | -207.6463 |
| | | Total: | 1285.2964 | 663.4914 | 1265 | 638.25 | |
| P-loss | = 20.29637 | MW | | | | | |
| Q-loss | = 202.9637 | MVAr | | | | | |

TABLE II
GENERATOR REACTIVE POWER CONTRIBUTIONS AT LOAD-BUSES AND Q-LOSS ALLOCATION FOR SAMPLE 5-BUS SYSTEM (PEAK-LOAD

| Load bus | Net demand (MW, MVAr) | Generator sources (MVAr) Gen.(G1) | Gen.(G2) | Total (MVAr) | Q-loss (MVAr) |
|---|---|---|---|---|---|
| 3 | 402.5, 161.18 | 141.53 (72%) | 55.406 (28%) | 196.93 | 35.756 |
| 4 | 345, 129.2 | 70.218 (39%) | 111.49 (61%) | 181.71 | 52.508 |
| 5 | 517.5, 207.65 | 167.71 (52%) | 154.64 (48%) | 322.35 | 114.7 |
| Total | 1265, 498.03 | 379.45 (54%) | 321.54 (46%) | 700.99 | 202.96 |

It can be seen from Table II that the allocation of reactive power loss (Q-loss) to bus-5 is more as compared with other two buses. Since bus-5 is situated at equal distance from two generator sources, to feed its demand, it contributes to more Q-loss in the lines. The reactive power contribution at bus-5 from G1 is 52% and that from G2 is 48% of the total reactive power contributions at that bus. The Q-loss allocated to bus-4 is more than that allocated to bus-3 even though there is higher load demand at bus-3. This is because, bus-3 is just half the distance away from G1 as compared to the distance of bus-4 with respect to G2. Consequently, the power has to flow double the distance from the nearby generator G2 to meet the load demand at bus-4. Hence, it contributes to more reactive Q-loss in the system. Moreover, the load-buses 3 and 4 are nearer to generator buses 1 and 2 respectively. Therefore, the reactive power requirements at these buses are met maximum by its nearest generator buses as seen from Table II. It can be clearly noted from the table that bus-5 is the weakest bus in the system.

We know that as loading on the system increases, the power loss taking place in the transmission system also increases. Therefore, the loss allocation to load-buses must increase as loading condition on the system moves from light load to peak load. For demonstration of the effectiveness of the Q-loss allocated at load-buses and its impact on system voltage stability, the loading on the system is increased step by step upto the maximum loadability point considering the Q-limits of the generators. The load demand at the buses are increased proportional to their initial base-case load levels, step-by-step to reach the maximum loadability point using a loading parameter. Similarly, the generator output is also increased in order to meet the increased load. Correspondingly, Q-loss allocated to load-buses is calculated. Fig. 3 shows the Q-loss allocated to load-buses for various load conditions upto the system maximum loadability point. The Q-loss allocated at load-buses for the different loading points including the system maximum loadability is shown in Table III. Identification of weak buses in the system can be done using this information and the result is shown in Table IV. The voltage magnitude at load-buses from the continuation power flow method is shown in Fig. 4. From Fig. 3, it can be noted that the Q-loss allocated to load-bus-5 is increasing drastically (in a highly exponential way) as the loading on the system is approaching towards the critical loading point. Similarly, the voltage magnitude at load-bus-5 is also uniformly decreasing as the system is moving towards the critical (maximum) loading point. However, at one operating point, it can be noted that the changes in load-bus voltages are not uniform as compared with that from other operating conditions because of hitting the reactive power limits of the generator. It can also be observed that the large change in Q-loss or voltage at load-bus-5 is not only observed around the critical loading point, but also for other loading conditions of the system as well (See Figs. 3 and 4). A comparative study is also done with the continuation power flow method [22] and results of the comparison are shown in Tables V and VI. From the proposed approach as well as other existing method, bus-5 has been identified as the weakest bus in the system.

TABLE III
ALLOCATION OF Q-LOSS AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS

| Load buses | Q-loss allocation in MVAr | | | | |
|---|---|---|---|---|---|
| | —- | Peak load (in % age) | | | |
| | Base-case | 20% | 25% | 30% | Max. |
| 3 | 25.5332 | 38.014 | 41.5643 | 46.7019 | 101.8204 |
| 4 | 38.4799 | 59.1575 | 65.2596 | 73.8075 | 139.5763 |
| 5 | 82.6038 | 127.3612 | 140.6154 | 139.5763 | 308.213 |

From these discussions, it can be inferred that the Q-loss

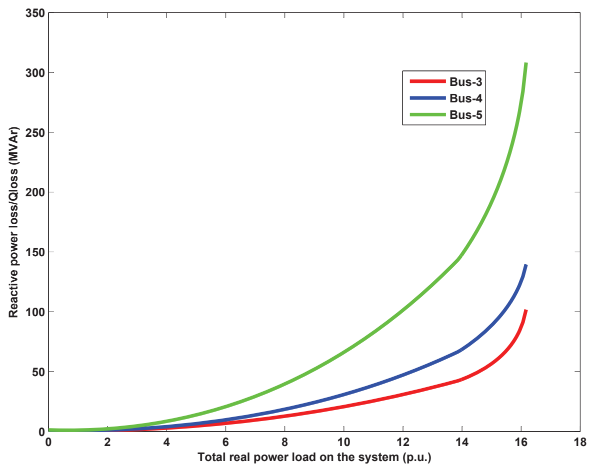


Fig. 3. Q-loss allocated at load-buses for variable load conditions upto critical loading point for sample 5-bus system

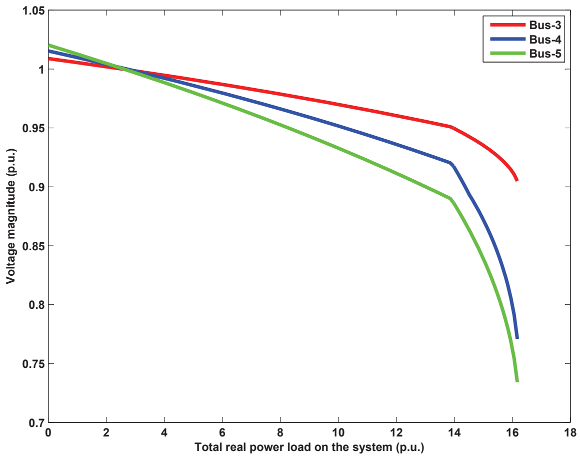


Fig. 4. Voltage profile at load-buses for various load conditions upto critical loading point of sample 5-bus system

TABLE IV
WEAK BUSES ORDER BASED ON REACTIVE POWER LOSS ALLOCATED AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS

| Weak buses order | | | | |
|---|---|---|---|---|
| —- | Peak load (in % age) | | | |
| Base-case | 20% | 25% | 30% | Max. |
| 5 | 5 | 5 | 5 | 5 |
| 4 | 4 | 4 | 4 | 4 |
| 3 | 3 | 3 | 3 | 3 |

TABLE V
VOLTAGE MAGNITUDE AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS FROM CPF

| Load buses | Voltage magnitude (p.u.) | | | | |
|---|---|---|---|---|---|
| | —- Peak load (in % age) | | | | |
| | Base-case | 20% | 25% | 30% | Max. |
| 3 | 0.9653 | 0.9544 | 0.9515 | 0.9455 | 0.9049 |
| 4 | 0.944 | 0.926 | 0.9213 | 0.903348 | 0.77091 |
| 5 | 0.9224 | 0.8979 | 0.8914 | 0.8728 | 0.7342 |

TABLE VI
WEAK BUSES ORDER BASED ON VOLTAGE MAGNITUDE AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS FROM CPF

| Weak buses order | | | | |
|---|---|---|---|---|
| —- | Peak load (in % age) | | | |
| Base-case | 20% | 25% | 30% | Max. |
| 5 | 5 | 5 | 5 | 5 |
| 4 | 4 | 4 | 4 | 4 |
| 3 | 3 | 3 | 3 | 3 |

allocated at load-buses give a clear indication about the system reactive power issues, which in-turn indicates the system voltage instability problem (See Figs. 3 and 4). For the system under consideration, load-bus-5 is the weakest bus followed by load-bus-4.

### B. 10-bus practical equivalent system

A 10-bus, 400 kV practical equivalent system of Indian southern region power grid (See Fig. 5) is tested using the proposed approach [23]. The system has 3 generators, 12 TLs and 7 loads. Initial base-case loading on the system is 1386 MW and 675 MVAr.

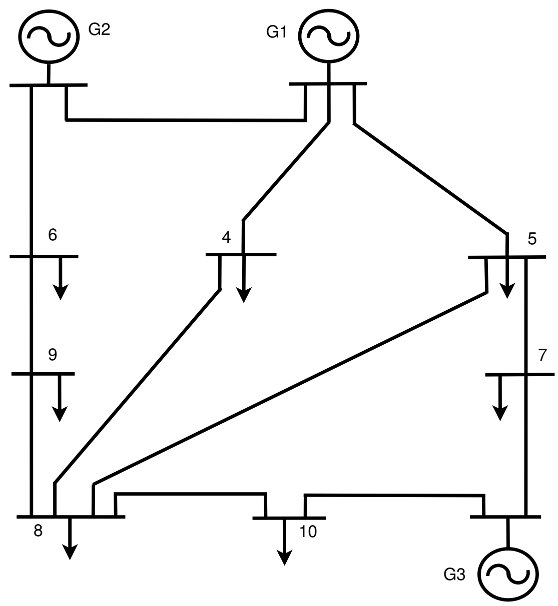


Fig. 5. Single-line diagram of 10-bus system

Similar to sample 5-bus system, the Q-loss allocated at load-buses is computed using the proposed approach upto the system maximum loadability point. Fig. 6 show the Q-loss allocated to load-buses for various load conditions upto the maximum loadability point. The magnitudes of load-bus voltages obtained using the continuation power flow method are also shown in Fig. 7.

For more clarity, the Q-loss allocated at load-buses for various load conditions is shown in Table VII. Based on the reactive power loss, the identified weak buses are shown in Table VIII. These results are compared with that of continuation power flow method as shown in Tables IX and X.

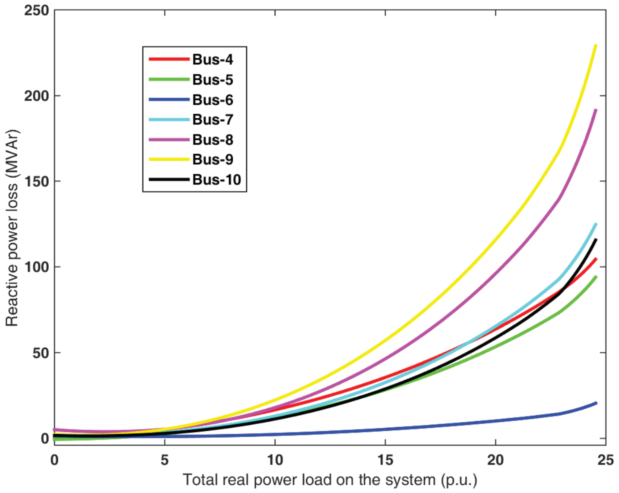


Fig. 6. Q-loss allocated at load-buses for various load conditions upto critical loading point for sample 10-bus system

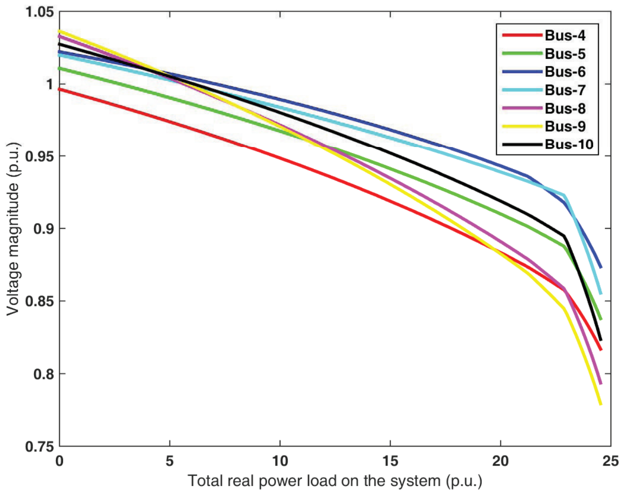


Fig. 7. Voltage profile at load-buses for various load conditions upto critical loading point

TABLE VII
Q-LOSS ALLOCATION AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS

| Load | Reactive power loss allocation (Q-loss) in MVAr | | | | |
|---|---|---|---|---|---|
| buses | —- | Peak load (in % age) | | | |
| | Base-case | 20% | 25% | 30% | Max. |
| 4 | 30.5095 | 43.6052 | 47.3404 | 51.2746 | 104.4354 |
| 5 | 23.881 | 35.4048 | 38.7286 | 42.2446 | 93.9699 |
| 6 | 4.4022 | 6.6252 | 7.2757 | 7.9649 | 20.4779 |
| 7 | 27.1159 | 41.6366 | 45.9278 | 50.5089 | 124.6622 |
| 8 | 38.2878 | 59.9191 | 66.4001 | 73.3553 | 191.4023 |
| 9 | 47.3839 | 73.2445 | 80.9161 | 89.1218 | 229.0071 |
| 10 | 23.9087 | 37.006 | 40.8999 | 45.0679 | 115.6414 |

TABLE VIII
WEAK BUSES ORDER BASED ON Q-LOSS ALLOCATED AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS

| Weak buses order | | | | |
|---|---|---|---|---|
| —- | Peak load (in % age) | | | |
| Base-case | 20% | 25% | 30% | Max. |
| 9 | 9 | 9 | 9 | 9 |
| 8 | 8 | 8 | 8 | 8 |
| 4 | 4 | 4 | 4 | 7 |
| 7 | 7 | 7 | 7 | 10 |
| 10 | 10 | 10 | 10 | 4 |
| 5 | 5 | 5 | 5 | 5 |
| 6 | 6 | 6 | 6 | 6 |

TABLE IX
VOLTAGE MAGNITUDE AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS FROM CPF

| Load | Voltage magnitude (p.u.) | | | | |
|---|---|---|---|---|---|
| buses | —- | Peak load (in % age) | | | |
| | Base-case | 20% | 25% | 30% | Max. |
| 4 | 0.9259 | 0.9079 | 0.9031 | 0.8982 | 0.8169 |
| 5 | 0.9475 | 0.9316 | 0.9273 | 0.923 | 0.8379 |
| 6 | 0.9735 | 0.9607 | 0.9573 | 0.9537 | 0.8736 |
| 7 | 0.9679 | 0.9554 | 0.9522 | 0.9488 | 0.8555 |
| 8 | 0.9435 | 0.9212 | 0.9153 | 0.9092 | 0.7936 |
| 9 | 0.9401 | 0.9157 | 0.9092 | 0.9026 | 0.7793 |
| 10 | 0.9588 | 0.9417 | 0.9372 | 0.9326 | 0.8236 |

TABLE X
WEAK BUSES ORDER BASED ON VOLTAGE MAGNITUDE AT LOAD-BUSES FOR VARIOUS LOAD CONDITIONS FROM CPF

| Weak buses order | | | | |
|---|---|---|---|---|
| —- | Peak load (in % age) | | | |
| Base-case | 20% | 25% | 30% | Max. |
| 4 | 4 | 4 | 4 | 9 |
| 9 | 9 | 9 | 9 | 8 |
| 8 | 8 | 8 | 8 | 4 |
| 5 | 5 | 5 | 5 | 10 |
| 10 | 10 | 10 | 10 | 5 |
| 7 | 7 | 7 | 7 | 7 |
| 6 | 6 | 6 | 6 | 6 |

It can be observed from the Fig. 6, and Tables VII-VIII that based on reactive power loss, the weakest bus in the system is bus-9 followed by bus-8. Moreover, we notice that the weakest load-buses (bus-9 and bus-8) produced by the proposed approach are the same for all loading conditions in the system unlike from continuation power flow method (Refer Table X and Fig. 7).

From these discussions, it can be inferred that using the continuation power flow method the identification of actual weakest load-buses is only possible when the system is under maximum loadabilty point. But, the iterative nature of the whole process seems to be computationally more expensive. However, using the proposed approach the weakest load-buses information is obtained from the initial base-case loading condition itself.

## IV. Conclusion

In this paper, the impact of reactive power loss allocated at load-buses on system voltage stability is presented. The reactive power loss allocations at each of the load-buses is determined using modified Y-bus method. This procedure is repeated for various load conditions in the system upto the system maximum loadability point. Further, this information is used for identifying the weak load-buses in the system. The proposed approach is applied on 5-bus and 10-bus systems. Comparative analysis of the proposed approach with the continuation power flow method is also carried out. The merit of this approach is that the weakest load-buses information can be obtained from the initial base-case loading condition of the system itself unlike from the continuation power flow method. Simulation results show that the reactive power loss allocated at load-buses give a clear indication about the system reactive power issues, which in-turn indicates the system voltage instability problem. Hence, this could be useful in control center for real time monitoring of the system against the voltage instability. Consequently, the system operator may plan and take preventive measures in order to maintain the system operates under reliable and secure conditions.

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