An Algorithm Steps to Solve Coupled Case for Dual Input Dual Output SCC

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Abstract—The proposed converter is designed for low power applications. In this paper, the algorithm is proposed to solve the coupled case of dual input and the dual output converter. The major contribution is R-parameters calculation for the coupled case is deliberated in detail where it includes all conduction and ohmic losses accounting for coupling effects. To validate the performance of designed SCC, modeling and mathematical analysis has been carried out. The results are verified using PSIM simulations and validated mathematically. The analytical and simulation results give excellent proof for the newly designed coupled converter.

Index Terms—SCC, DIDO, low power, algorithm, coupled, dual input, dual output.

I. INTRODUCTION

A conventional power management (CPM) IC contains both front end and back end DC-DC converter connections. Linear regulators and low drop out regulators are used to provide multiple output voltages [1], [2]. In order to reduce power losses both front end and back-end DC-DC converter need to be efficient. The main disadvantage of CPMs are more weight and excess consumption of space [3], [4]. Due to more space consumption, all components need to be implemented using integrated circuits called as PMICs. Recently, switched capacitor converter (SCC) is designed which includes all the components such as drivers, gates, controllers and sensing circuits are designed and wrapped in silicon circuits [5]. Portable electronic devices requires less power consumptions which can be achieved by designing a multiple output SCC and is used for different blocks in electronic circuits [6]. Dual input dual output (DIDO) SCC is used for low power applications by providing continuous power. SCCs are developed by many researchers [4], [5], [7] and the advantages of SCCs are, non magnetic elements, and the ease of integration into ICs. Furthermore, SCC has high efficiency in open loop conditions for fixed input and fixed output voltage conversion ratios (VCRs) [5], [6]. Hence, portable applications preferred SCC for high efficiency, low efficiency drop out for the technological processors [7], [8]. Major area of SCC studies is implementation of maximum number of VCRs with less number of switches and capacitors. Employing more number of switches and capacitor elements may result in higher losses, and their reduced efficiency.

To overcome the issue, DIDO converter is proposed and advantage of which provides less size and area efficiency for on chip implementation. Chia-Min Chen et al. [9] have designed an on chip dual output converter with certain limits. The $1^{\rm st}$ output of the converter provides mere boost voltage with respect to input voltage, whereas the $2^{\rm nd}$ output generates buck voltage alone. The converter is designed with no cross regulation effect. Zhaikhan et al. [6] designed a DIDO SCC for 32 voltage conversion ratios (VCRs), i.e. multiple output voltages. Among 32 VCRs some of the VCRs have coupled conditions. According to Zhaikhan et al. [6], a coupled case or cross regulation effect, equivalent resistance $(R_{\rm eq})$ are difficult to design and analyse respectively.

This paper explains R-parameters calculation for coupled case DIDO SCC by two-port system modeling of SCC. The primary advantages of proposed converter are $(V_{\rm o1})$ and $(V_{\rm o2})$ provide buck, boost and unity voltages.

II. THE CHALLENGE OF COUPLED CASE

Cross-coupling is one of the major issues of dual output converter but for proposed converters it is an issue for only certain number of VCRs. For low power and low current converters coupling effect will be too small so it is negligible. If current exceeds 100 mA, coupling effect may be present in the converter [6]. In this paper, *coupling effects are included*

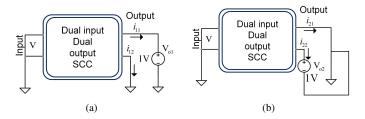


Fig. 1: General two port system.

and derived analytically. Equivalent model R-parameters are calculated analytically. Analytical solution for R-parameters are in good coincidence with simulation results.

III. VOLTAGE SOURCE METHOD OF R-PARAMETERS ${\sf CALCULATION}$

New approach for finding $R_{\rm eq}$ proposes to treat dual output ternary SCC as a two-port system shown in Fig. 1. Analysis will be based on resistance and transresistance parameters (R-parameters). According to the two-port system analysis, output

voltages for load resistances R_1 and R_2 are described as in (1), (2).

$$V_{o1} = V_{TR1} - I_{L1}R_{11} - I_{L2}R_{12}; (1)$$

$$V_{o2} = V_{TR2} - I_{L1}R_{21} - I_{L2}R_{22}, \tag{2}$$

where load currents I_{L1} and I_{L2} can be written as:

$$I_{L1} = \frac{V_{01}}{R_1}; \qquad I_{L2} = \frac{V_{02}}{R_2}.$$
 (3)

In (1), (2), V_{o1} , V_{o2} are actual output voltages, while V_{TR1} and V_{TR2} are targeted output voltages. Therefore, the dual output SCC model (4) and (5) becomes a function of resistance and transresistance parameters, which constitute for R_{eq} (R-parameters) of the converter. R_{11} and R_{22} account for normal equivalent resistances, while R_{12} and R_{21} are for coupling resistances (transresistances).

$$V_{01} = \frac{V_{TR1}R_1(R_2 + R_{22}) - V_{TR2}R_1R_{12}}{(R_{11} + R_1)(R_{22} + R_2) - R_{12}R_{21}};$$
(4)

$$V_{02} = \frac{V_{TR2}R_2(R_1 + R_{11}) - V_{TR1}R_2R_{21}}{(R_{11} + R_1)(R_{22} + R_2) - R_{12}R_{21}}.$$
 (5)

To find unknown R-parameters, we need to apply the following test:

- 1) Short-circuit all input voltage sources.
- 2) Connect 1 V voltage source (V) to the first output and short-circuit the second one as shown in Fig. 1(a).
- 3) At this state, find the average load currents I_{11} and I_{21} .
- 4) Repeat steps 1 and 2 for finding I_{12} and I_{22} . However, in this case, short-circuit the first output and replace the second load with 1 V voltage source as shown in Fig. 1(b).

Above procedure yields I-matrix of the two-port system. Since Y=I/V and V=1~V, I-matrix will simply be equal to Y-matrix. Inverse of Y-matrix will finally bring to R-matrix (R-parameters). More detailed calculations are shown in Section V.

It should be noted that this method neglects the effect of filter (load) capacitors. As a result, theoretical calculations are less complex. For simulation and experimental measurements of R-parameters, it is suggested to use current sources and to measure average output voltages to account for filter capacitors effects.

IV. CIRCUIT DESCRIPTION OF DIDO [6]

The proposed SCC configuration is illustrated in Fig. 2. It is basically a two capacitor series-parallel SCC, modified with DIDO converter. It can generate 36 VCRs (coupled and decoupled) with limited number of switches and capacitors. In total, 2 capacitors and 11 switches are used. All decoupled case and coupled case VCRs are listed in Zhaikhan et al. [6]. Coupled VCRs are shown in Table I [6]. Main principle of operation is based on two phase (parallel or series) charging and discharging of flying capacitors. In dual output, the number of phases is doubled so that each of the output VCR implementation is characterized with separate charging and discharging states respectively. Separate implementation of

each output VCR reduces the possibility of cross coupling effects [10]. $R_{\rm eq}$ detailed schematic is depicted in Fig. 3 which is used for solving R-parameters analysis and is discussed in upcoming Section V.

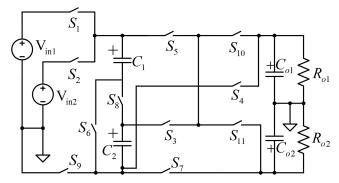


Fig. 2: Proposed dual input dual output SC converter [6].

TABLE I: Coupled VCRs

$V_{o1} = V_{in2}$ and $V_{o2} = (0.5 * V_{in1}) + V_{in2}$
$V_{o1} = (2 * V_{in1})$ and $V_{o2} = (1.5 * V_{in1})$
$V_{o2} = (2 * V_{in2})$ and $V_{o1} = (2 * V_{in1}) + (2 * V_{in2})$
$V_{o1} = (2 * V_{in2}) \text{ and } V_{o2} = V_{in2}$
$V_{o2} = V_{in2}$ and $V_{o1} = (0.5 * V_{in1}) + V_{in2}$
$V_{o1} = (0.5 * V_{in1}) + (0.5 * V_{in2})$ and $V_{o2} = 0.5 * V_{in}$
$V_{o2} = (2 * V_{in1}) \text{ and } V_{o1} = (1.5 * V_{in1})$
$V_{o1} = (2 * V_{in2}) \text{ and } V_{o2} = (2 * V_{in1}) + (2 * V_{in2})$
$V_{o2} = (2 * V_{in2})$ and $V_{o1} = V_{in2}$

V. Steps to be followed for solving coupled case

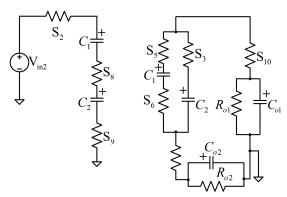
The R-parameters are derived using following analytical process, i.e. RC network and partial KVL methods. For simplicity, $V_{\rm o1} = (0.5*V_{\rm in1}) + (0.5*V_{\rm in2})$ and $V_{\rm o2} = 0.5*V_{\rm in1}$ VCR are considered for upcoming R-parameters analysis. Some of the assumptions and solutions are as follows:

- Equal switch resistance (r) and flying capacitor capacitance (C) are considered.
- 2) Filter (load) capacitors (C_{o1} and C_{o2}) effect is neglected.
- ESR of flying capacitors is not considered in calculations. However, this parameter can be easily included if required.

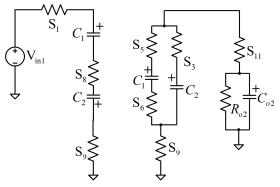
From Fig. 4, phase 1 (Fig. 4(a)) and phase 3 (Fig. 4(c)) are the first order RC-circuits, while phase 2 (Fig. 4(b)) and phase 4 (Fig. 4(d)) are the higher order ones. Therefore, phase 1 and 3 have one time constant (T_1 and T_4), whereas phase 2 and 4 - two time constants (T_2 and T_3 ; T_5 and T_6). In addition, time constant values for phase 1 and 3 (as well as phase 2 and 4) are similar due to the symmetry.

$$T_1 = \frac{3}{2}rC; \ T_2 = 1.438rC; \ T_3 = 5.562rC;$$

 $T_4 = \frac{3}{2}rC; \ T_5 = 1.438rC; \ T_6 = 5.562rC.$ (6)



(a) $V_{\rm o1}$ equivalent circuit: charging and discharging



(b) V_{o2} equivalent circuit: charging and discharging

Fig. 3: Equivalent circuit of VCR V_{o1} =(0.5* V_{in1})+(0.5* V_{in2}) and $V_{o2}=0.5*V_{in1}.$

DC voltage source (V) is connected at the output terminal $(V_{\rm o1})$ when the input voltages $(V_{\rm in1}$ and $V_{\rm in2})$ and $V_{\rm o2}$ are shorted with respect to ground. Capacitor voltage expression for each phase is given by,

$$V_{1C_{12}} = V_{1C_{12}}(0)e^{-\frac{t}{T_1}}; (7)$$

$$V_{2C_1} = V + a_{11}e^{-\frac{t}{T_2}} + a_{12}e^{-\frac{t}{T_3}}; (8)$$

$$V_{2C_2} = V + a_{21}e^{-\frac{t}{T_2}} + a_{22}e^{-\frac{t}{T_3}}; (9)$$

$$V_{3C_{12}} = V_{3C_{12}}(0)e^{-\frac{t}{T_4}}; (10)$$

$$V_{4C_1} = a_{31}e^{-\frac{t}{T_5}} + a_{32}e^{-\frac{t}{T_6}}; (11)$$

$$V_{4C_2} = a_{41}e^{-\frac{t}{T_5}} + a_{42}e^{-\frac{t}{T_6}}. (12)$$

To find 10 unknown coefficients, a set of 10 linear equations should be constructed. The 1st four equations are from capacitor voltage continuity principle and they are given by,

$$V_{1C_{12}}\left(\frac{T}{4}\right) = V_{2C_1}(0) + V_{2C_2}(0); \tag{13}$$

$$V_{2C_1}\left(\frac{T}{4}\right) + V_{2C_2}\left(\frac{T}{4}\right) = V_{3C_{12}}(0);$$
 (14)

$$V_{3C_{12}}\left(\frac{T}{4}\right) = V_{4C_1}(0) + V_{4C_2}(0); \tag{15}$$

$$V_{4C_1}\left(\frac{T}{4}\right) + V_{4C_2}\left(\frac{T}{4}\right) = V_{1C_{12}}(0).$$
 (16)

Similarly, the other two equations are from charge conservation between two capacitors plates (C_1 and C_2 in phase 1 and phase 3) are given by,

$$C_1 V_{2C_1} \left(\frac{T}{4} \right) - C_2 V_{2C_2} \left(\frac{T}{4} \right) = C_1 V_{4C_1}(0) - C_2 V_{4C_2}(0);$$
(17)

$$C_1 V_{4C_1} \left(\frac{T}{4} \right) - C_2 V_{4C_2} \left(\frac{T}{4} \right) = C_1 V_{2C_1}(0) - C_2 V_{2C_2}(0).$$
(18)

The remaining two equations each are derived respectively from phase 2 and phase 4 using partial KVL (only exponential terms) which are given in (19) and (20)

$$\left(\frac{R_2C_1}{T_2}a_{11}e^{-\frac{t}{T_2}} + \frac{R_2C_1}{T_3}a_{12}e^{-\frac{t}{T_3}}\right) - a_{11}e^{-\frac{t}{T_2}} - a_{12}e^{-\frac{t}{T_3}};$$

$$= \left(\frac{R_3C_2}{T_2}a_{21}e^{-\frac{t}{T_2}} + \frac{R_3C_2}{T_3}a_{22}e^{-\frac{t}{T_3}}\right) - a_{21}e^{-\frac{t}{T_2}} - a_{22}e^{-\frac{t}{T_3}};$$

$$\left(19\right)$$

$$\left(\frac{R_6C_1}{T_5}a_{31}e^{-\frac{t}{T_5}} + \frac{R_6C_1}{T_6}a_{32}e^{-\frac{t}{T_6}}\right) - a_{31}e^{-\frac{t}{T_5}} - a_{32}e^{-\frac{t}{T_6}};$$

$$= \left(\frac{R_7C_2}{T_5}a_{41}e^{-\frac{t}{T_5}} + \frac{R_7C_2}{T_6}a_{42}e^{-\frac{t}{T_6}}\right) - a_{41}e^{-\frac{t}{T_5}} - a_{42}e^{-\frac{t}{T_6}}.$$
(20)

Once the coefficients are identified from (13) - (20) the Y-parameters $(Y_{11} \text{ and } Y_{21})$ need to be calculated using the ratio between average current that are flowing through each load $(I_{11} \text{ and } I_{21})$ to source voltage (V) which is given by,

$$Y_{11} = \frac{I_{11}}{V}; \quad Y_{21} = \frac{I_{21}}{V}.$$
 (21)

Similarly, source voltage (V) is applied at the $V_{\rm o2}$ and the $V_{\rm in1}, \, V_{\rm in2}, \, V_{\rm o2}$ are shorted with respect to ground. Capacitor voltage expression for each phase is given by,

$$V_{1C_{12}} = V_{1C_{12}}(0)e^{-\frac{t}{T_1}}; (22)$$

$$V_{2C_1} = -V + a_{11}e^{-\frac{t}{T_2}} + a_{12}e^{-\frac{t}{T_3}}; (23)$$

$$V_{2C_2} = -V + a_{21}e^{-\frac{t}{T_2}} + a_{22}e^{-\frac{t}{T_3}}; (24)$$

$$V_{3C_{12}} = V_{3C_{12}}(0)e^{-\frac{t}{T_4}}; (25)$$

$$V_{4C_1} = V + a_{31}e^{-\frac{t}{T_5}} + a_{32}e^{-\frac{t}{T_6}}; (26)$$

$$V_{4C_2} = V + a_{41}e^{-\frac{t}{T_5}} + a_{42}e^{-\frac{t}{T_6}}. (27)$$

Similarly, for another case the unknown coefficients, a set of 10 linear equations and Y-parameters are calculated by following the same procedure as derived in the previous steps. Due to reciprocal relation between resistance and conductance parameters, R-parameters can be found as,

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}.$$
 (28)

Finally, the simplified R-parameters calculation for R-parameters are given in (29) - (31),

$$R_{11} = -\frac{\left[0.015*exp\left(\frac{0.341}{C*f*r}\right) + 0.985*exp\left(\frac{0.174}{C*f*r}\right) + 0.015*exp\left(\frac{0.045}{C*f*r}\right) - 1.0*exp\left(\frac{0.385}{C*f*r}\right) + 0.985*exp\left(\frac{0.212}{C*f*r}\right) - 1.0\right]}{C*f*\left[exp\left(\frac{0.167}{C*f*r}\right) - 1.0*exp\left(\frac{0.341}{C*f*r}\right) + exp\left(\frac{0.174}{C*f*r}\right) + exp\left(\frac{0.045}{C*f*r}\right) + exp\left(\frac{0.385}{C*f*r}\right) - 1.0*exp\left(\frac{0.212}{C*f*r}\right) - 1.0*exp\left(\frac{0.219}{C*f*r}\right) - 1.0\right]};$$

$$(29)$$

$$R_{21} = R_{12} = -\frac{9.701*10^{-5}*\left[77*exp\left(\frac{0.341}{C*f*r}\right) + 5077*exp\left(\frac{0.174}{C*f*r}\right) + 77*exp\left(\frac{0.045}{C*f*r}\right) - 5154*exp\left(\frac{0.385}{C*f*r}\right) + 5077*exp\left(\frac{0.212}{C*f*r}\right) - 5154\right]}{C*f*\left[exp\left(\frac{0.167}{C*f*r}\right) - 1.0*exp\left(\frac{0.341}{C*f*r}\right) + exp\left(\frac{0.174}{C*f*r}\right) + exp\left(\frac{0.045}{C*f*r}\right) + exp\left(\frac{0.385}{C*f*r}\right) - 1.0*exp\left(\frac{0.212}{C*f*r}\right) - 1.0*exp\left(\frac{0.219}{C*f*r}\right) - 1.0\right]};$$

$$(30)$$

$$R_{22} = -\frac{151.6*[0.127*exp(\frac{0.507}{C*f*r}) - 0.127*exp(\frac{0.264}{C*f*r}) - 0.250*exp(\frac{0.552}{C*f*r}) - 0.127*exp(\frac{0.597}{C*f*r}) - 0.002*exp(\frac{0.681}{C*f*r}) + }{C*f*[2577*exp(\frac{0.167}{C*f*r}) + 2500*exp(\frac{0.314}{C*f*r}) + 2500*exp(\frac{0.174}{C*f*r}) - 2500*exp(\frac{0.045}{C*f*r}) - 2577*exp(\frac{0.385}{C*f*r}) - 2500*exp(\frac{0.212}{C*f*r}) - 2577*exp(\frac{0.212}{C*f*r}) - 2577*exp(\frac{0.212}{C*f*r})$$

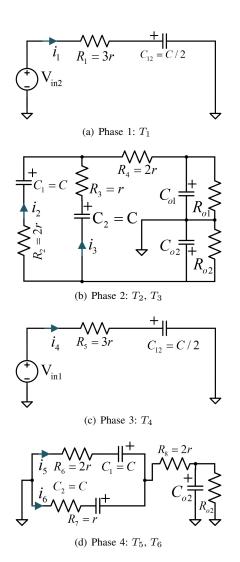


Fig. 4: Different phases of V_{o1} =(0.5 * V_{in1})+(0.5 * V_{in2}) and $V_{o2}=0.5*V_{in1}$ VCRs.

VI. MEASUREMENTS AND DISCUSSION

Simulation and parameter selection of proposed converter is discussed in this section. For simplicity, analysis is performed for only one VCR ($V_{\rm o1}$ =(0.5 * $V_{\rm in1}$)+(0.5 * $V_{\rm in2}$) and $V_{\rm o2}$ = 0.5 * $V_{\rm in1}$). For modeling the coupled converter, equivalent resistance needs to be calculated and it is verified by PSIM simulation. Simulation parameters strictly follow to the

TABLE II: Parameter for dual input dual output converter

SI.No	Parameters	Quantity		
1	MAX4678 switch $(S_1 - S_{11})$	11		
2	Flying capacitor(C_1 and C_2)	22 μF		
3	Output capacitor(C_{o1} and C_{o2})	220 μF		
4	Load resistance (R_{o1} and R_{o2})	Variable 200 Ω to 200 K Ω		
5	ESR $(C_1 \text{ and } C_2)$	100 mΩ		
6	Switch on resistance (r_{on})	0.3 Ω		
7	Input voltage Range (V_{s1} and V_{s2})	1 V to 5 V		
8	Output voltage Range (V_{o1} and V_{02})	0.1 V to 10 V		

TABLE III: Comparison results of R-parameters with different frequencies

Frequency	Parameter	R_{11}, Ω	R_{21}, Ω	R_{12}, Ω	R_{22}, Ω
100 kHz	Modelling	9.210	4.605	4.605	4.018
	Simulation	9.201	4.599	4.599	4.015
50 kHz	Modelling	9.241	4.621	4.621	4.046
	Simulation	9.234	4.621	4.616	4.043

circuit ratings described in this Table II. Fig. 5(a) and Fig. 5(b) illustrate the simulation output voltages of the proposed VCR (V_{o1} =(0.5 * V_{in1})+(0.5 * V_{in2}) and V_{o2} = 0.5 * V_{in1}) with different frequency. From Table III it is clear that modelled and simulated results are in good agreement to verify the coupled case of proposed converter.

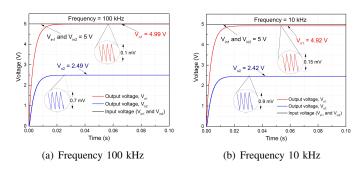


Fig. 5: Simulation result of VCR V_{o1} =(0.5 * V_{in1})+(0.5 * V_{o1} and $V_{o2}=0.5$ * $V_{o1}.$

VII. CONCLUSION

Proposed dual input/output SCC can implement 32 VCRs using 11 switches and 2 capacitors. The mathematical modeling was implemented only for decoupled cases of VCRs in [6] due to no currently existing model for coupling effect. This paper presents R-parameters analysis for coupled cases of dual output SCC implementation. Coupling was analyzed by new

approach which treats dual output SCC as two-port system. Cross regulation parameters were calculated and validated with simulations and mathematical proof. Future scope of the work is validating the coupled case, experimentally using DIDO converter.

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