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# Buckling and Free Vibration Behavior of a Temperature Dependent FG-CNTRC Cylindrical Panel Under Thermal Load

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## Abstract

Present study deals with the buckling and free vibration behavior of functionally graded carbon-nanotubes reinforced composite (FG-CNTRC) cylindrical panel exposed to uniform thermal load. Stresses setup due to thermal load and temperature dependent properties influences the buckling and free vibration behavior of the heated structures. Approach employed in the present study consists of static analysis to compute thermal stresses, eigen-value buckling analysis to compute critical buckling temperature and finally modal analysis, taking thermal stresses into account. Influence of different CNTs grading pattern, CNTs volume fraction, geometric parameters, boundary constraints and temperature dependent properties on the buckling strength are investigated. It is observed that hybrid CNTs distribution pattern gives comparatively higher buckling strength and free vibration frequencies. Investigation on free vibration characteristics of the FG-CNTRC panel at elevated temperature signifies that the decline in free vibration frequencies is very drastic at a temperature close to buckling temperature along with temperature dependent properties. © 2018 Elsevier Ltd. All rights reserved.

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*Keywords:* Cylindrical panel; Carbon-nanotubes; Finite element method; Thermal buckling; Free vibration.

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## 1. Introduction

Today, functionally graded carbon-nanotube reinforced composite (FG-CNTRC) materials are considered to be the replacement of conventional carbon fibers by carbon-nanotubes (CNTs). Carbon nanotubes (CNTs) has extraordinary physical and mechanical properties thus considered as a potential candidate for reinforcement of polymer composites [1]. CNT reinforced polymers exhibit high specific strength and stiffness thus viewed as an emerging material in the 21st century [2]. [3] found that the combination of FGMs and CNTs can be achieved by nonuniform distribution of CNTs across the thickness of composite in a specific way. Later, [4] found that powder metallurgy process can be utilized to prepare the functionally graded CNT-reinforced aluminum matrix composites. Looking at the potential of FG-CNTRC in the wide area of engineering applications, study on its failure mechanism has received substantial attention. Thin cylindrical panel-like structures employed in high-speed aircraft, rockets, reactor vessels, turbines,

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and spacecraft, when in service are exposed to the severe thermal environment. This thermal environment develops stresses in the structures which result in thermal buckling and also affects the free vibration behavior of the panel. Thus, buckling and free vibration characteristics of a FG-CNTRC cylindrical panel under thermal load are considered to be the important factor in the design stages.

Several researchers have investigated the buckling strength of cylindrical panels under thermal load. [5] employed first order shear deformation theory (FSDT) and element-free kp-Ritz method to analyze the buckling behavior of FG-CNTRC cylindrical panel under thermal and mechanical load. Later, [6] used higher order shear deformation theory (HSDT) to investigate the buckling strength of FG-CNTRC cylindrical shells under uniform temperature rise. Recently, differential quadrature method was used by [7] to analyze the buckling strength of FG-CNTRC panel under uniform and variable load. Vibration behavior of FG-CNTRC cylindrical panels was studied by [8] considering the three-dimensional theory of elasticity. They found that CNTs distribution pattern and volume fraction of CNT influences the natural frequency significantly. Eshelby-Mori-Tanaka approach based micromechanical model was employed by [9] to estimate the effective material properties of FG-CNTRC. They analyzed free vibration behavior of FG-CNTRC cylindrical panels by implementing the element-free kp-Ritz method. [10] analyzed the flexural strength and free vibration of FG-CNTRC cylindrical panels using FSDT. Free vibration and buckling behavior of singly and doubly curved functionally graded shell panels under thermal and uni-axial compressive load was examined by [11] using HSDT. Effect of thermal load on the vibration behavior of the nanocomposite cylindrical shells was investigated by [12]. Later, [13], [14] and [15] employed the finite element tool to analyze the influence of thermal load on buckling and free vibration behavior of cylindrical panel. The review of the open literature disclosed that study on combined buckling and free vibration behavior of FG-CNTRC cylindrical panel with temperature dependent properties under uniform thermal load has not been investigated which is considered to be important from the practical viewpoint. The present investigation emphasizes on these aspects.

## 2. Modeling of material for FG-CNT Reinforced Composite

CNTRC cylindrical panel considered for the analysis is made from a mixture of single-walled carbon nanotubes (SWCNT) and isotropic matrix. The present study deals with eight variants of through thickness functionally graded distribution of carbon-nanotubes namely FG-X, FG-O, FG-V, FG-Λ, FG-V-O-Λ, FG-Λ-O-V, FG-V-X-Λ and FG-Λ-X-V as shown in Figs. 1-2. In the present study, the rule of mixtures approach is being used to compute the effective material properties of the reinforced composite panel. The effective shear modulus and Young’s modulus of the panel can be extracted by employing Eq. 1 ([16]).

$$\begin{aligned}
 E_{11} &= \eta_1 V_{CNT} E_{11}^{CNT} + V_m E^M \\
 \frac{\eta_2}{E_{22}} &= \frac{V_{CNT}}{E_{22}^{CNT}} + \frac{V_m}{E^M} \\
 \frac{\eta_3}{G_{12}} &= \frac{V_{CNT}}{G_{12}^{CNT}} + \frac{V_m}{G^M}
 \end{aligned}
 \tag{1}$$

In Eq. 1  $\eta_1$ ,  $\eta_2$  and  $\eta_3$  denotes efficiency parameters introduced to obtain the size dependent material properties of the FG-CNTRC cylindrical panel. Young modulus and shear modulus of SWCNTs are denoted by  $E_{11}^{CNT}$ ;  $E_{22}^{CNT}$  and  $G_{12}^{CNT}$ , respectively. Furthermore, properties of the isotropic matrix are given by  $E^M$  and  $G^M$ .  $V_{CNT}$  and  $V_m$  represents the volume fraction of CNTs and matrix respectively and follows the relation

$$V_{CNT} + V_m = 1
 \tag{2}$$

In the present study, FG-CNTRC panel with five different types of CNTs distributions ([17]) through the thickness directions has been analyzed along with four hybrid CNTs distributions. The volume fraction of CNTs in each lamina as a function of thickness coordinate for the four different types of CNTs distributions are as follows ([17]).

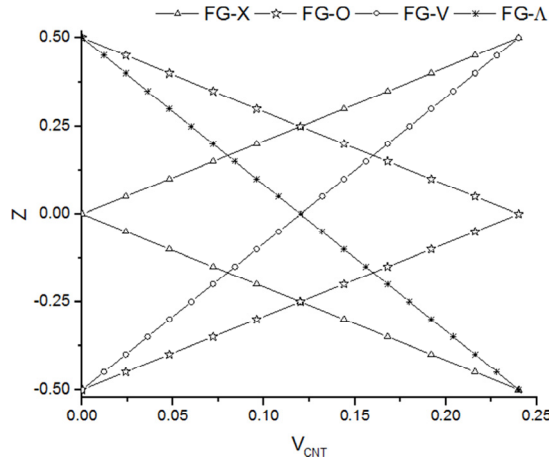


Fig. 1. CNTs grading patterns through thickness (FG-X, FG-O, FG-V and FG-Λ)

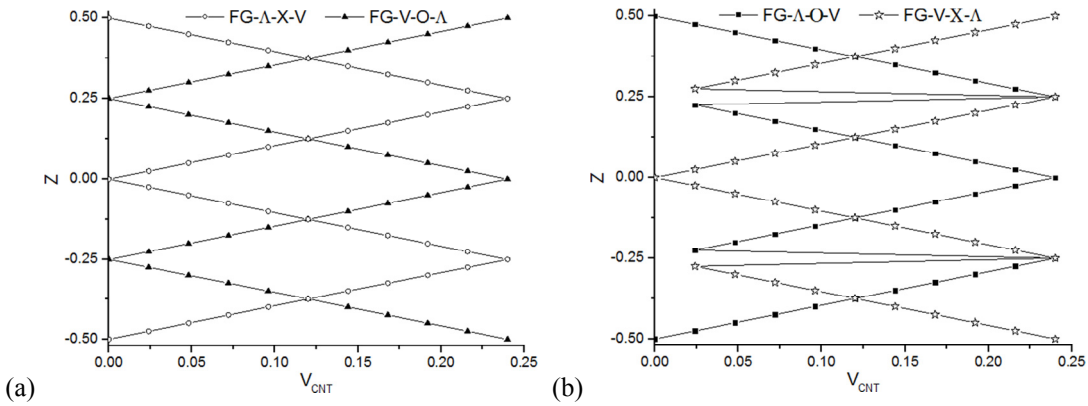


Fig. 2. Hybrid CNTs grading pattern through thickness (a) FG-Λ-X-V and FG-V-O-Λ (b) FG-Λ-O-V and FG-V-X-Λ

$$V_{CNT}(Z) = \begin{cases} 2\left(\frac{2|z|}{h}\right)V_{CNT}^* & (FG - X) \\ 2\left(1 - \frac{|2z|}{h}\right)V_{CNT}^* & (FG - O) \\ \left(1 + \frac{2z}{h}\right)V_{CNT}^* & (FG - V) \\ \left(1 - \frac{2z}{h}\right)V_{CNT}^* & (FG - \Lambda) \end{cases} \quad (3)$$

Using above CNTs grading patterns, hybrid FG-CNT reinforced composites shown in Fig. 2 are developed.

$$V_{CNT}^* = \frac{W_{CNT}}{W_{CNT} + (\rho^{CNT}/\rho^m) - (\rho^{CNT}/\rho^m)W_{CNT}} \quad (4)$$

where, mass fraction and mass density of the CNTs are given by  $W_{CNT}$  and  $\rho^{CNT}$  respectively, whereas mass density of the isotropic matrix is given by  $\rho^m$ . The thermal expansion coefficients in longitudinal and transverse directions are denoted by  $a_{11}$  and  $a_{22}$ , respectively. The Poisson’s ratio  $\nu_{12}$  and the overall mass density  $\rho$  are as given below.

$$\nu_{12} = V_{CNT}\nu_{12}^{CNT} + V_m\nu^m \quad (5)$$

$$\rho = V_{CNT}\rho^{CNT} + V_m\rho^m \quad (6)$$

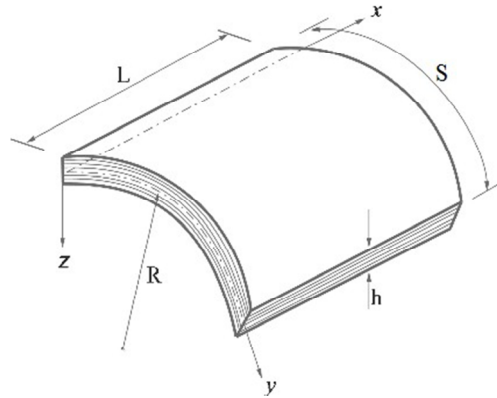


Fig. 3. Geometry of the Cylindrical panel

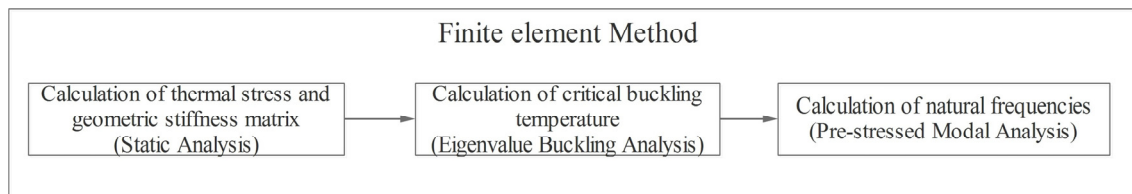


Fig. 4. A scheme of numerical analysis approach

$$\alpha_{11} = V_{CNT}\alpha_{11}^{CNT} + V_m\alpha^m \tag{7}$$

$$\alpha_{22} = (1 + \nu_{12}^{CNT})V_{CNT}\alpha_{22}^{CNT} + V_m\alpha^m - \nu_{12}\alpha_{11} \tag{8}$$

### 3. Analysis approach

Finite Element Method (FEM) has been employed to study the buckling and free vibration behavior of FG-CNTRC cylindrical panel. The model is discretized using an eight-node isoparametric shell element (Shell-281), available in ANSYS library. Shell 281 is an eight-noded element with six degrees of freedom at each node: three translation and three rotation is considered for finite element analysis. Commercially available finite element tool, ANSYS is used in the present study. The cylindrical panel analyzed with thickness (h), the radius of curvature (R), the length of the panel (L) and width of the panel (S) is shown in Fig. 3. The panel is investigated for different geometrical parameters such as curvature ratio(R/S), thickness ratio(S/h) and structural boundary constraints along with temperature dependent material properties.

#### 3.1. Finite element formulation

Stresses developed in the cylindrical panel because of the thermal load is computed by employing static analysis. Eigenvalue buckling analysis is carried out to estimate the critical buckling temperature. Further, to find the effect of thermal load on the natural frequencies, a pre-stressed modal analysis is carried out with critical buckling temperature as one of the parameters as shown in Fig. 4. Panel (for any boundary condition with edges restrained) will develop thermal stress when it is subjected to a temperature rise above ambient by  $\Delta T$ . Buckling analysis is implemented using the structural stiffness matrix  $[K]$  and geometric stiffness matrix  $[K_\sigma]$  based on the following relation.

$$([K] + \lambda_i[K_\sigma]) \{\psi_i\} = 0 \tag{9}$$

where  $\lambda_i$  is the eigenvalue and  $\{\psi\}$  is the corresponding eigenvector for  $i^{th}$  buckling mode. The product of the temperature rise  $\Delta T$  (above ambient temperature) and the lowest eigenvalue,  $\lambda_i$  gives the critical buckling temperature,  $T_{cr}$  that is  $T_{cr} = \lambda_1 \Delta T$ .

To determine buckling temperature for a material with temperature dependent properties an iterative numerical procedure given below is being employed.

1. Initially buckling temperature ( $\Delta T_{cr}$ ) is obtained for a panel with temperature-independent material properties at reference temperature  $T_0$  using Eq.9.
2. Update stiffness matrix  $[K]$  by changing the property values at  $T = T_0 + \Delta T_{cr}$  to evaluate a new buckling temperature.
3. Repeat step 2, till the thermal buckling temperature converges to a prescribed error tolerance.

In the iteration process, error tolerance is given by the relative difference between the two consecutive values.

$$\varepsilon = \left| \frac{\Delta T_{cr}^{(i+1)} - \Delta T_{cr}^{(i)}}{\Delta T_{cr}^{(i)}} \right| \leq 10^{-4} \quad (10)$$

Pre-stressed modal analysis is employed to predict the natural frequency of the pre-loaded structure. The natural frequency at any given temperature can be determined by using geometric stiffness matrix at that temperature and by solving the eigenvalue problem as given below.

$$([K] + [K_{\sigma}]) - \omega_k^2 [M] \{\phi_k\} = 0 \quad (11)$$

where,  $[M]$  is the structural mass matrix, while  $\omega_k$  is the natural frequency of the pre-stressed structure and  $\{\phi_k\}$  the corresponding mode shape. In the present study, commercial finite element software, ANSYS is employed to study the thermal buckling and free vibration characteristics of the FG-CNTRC cylindrical panel.

## 4. Validation studies

### 4.1. Critical buckling temperature

Thermal buckling strength of FGM cylindrical shell with temperature dependent properties investigated by [18] is considered for the validation. Material analyzed was Si3N4/SUS304 with proportionate dimensions of the panel are  $L = 300R_h$ ,  $R/h = 400$ . [18] employed hybrid fourier-GDQ method, while present method used SHELL 281 formulated based on FSDT. Critical buckling temperature determined using the present method shows close match with that of [18] as seen in Table 1.

Table 1. Comparison of buckling strength of the FGM panel with TD properties

| Volume fraction index | Critical buckling temperature |         |        |
|-----------------------|-------------------------------|---------|--------|
|                       | [18]                          | Present | %diff. |
| 0                     | 392.79                        | 393.95  | 0.29   |
| 0.2                   | 402.31                        | 401.86  | 0.11   |
| 0.5                   | 412.55                        | 411.09  | 0.35   |
| 1                     | 424.15                        | 421.61  | 0.60   |

### 4.2. Evaluation of Free Vibration Frequencies

[10] analyzed CNT reinforced cylindrical panel for its static and dynamic behavior which is considered for the validation. Dimension of the panel investigated are  $h=0.002m$ ,  $h/R=0.002$ ,  $L/R=0.1$  and  $\theta = 0.1rad$ . For comparison, two types of CNT distribution namely UD and FG-X are considered with 0.12 of CNT volume fraction. The present

method uses FEA tool whereas [10] used an approach based on first order shear deformation theory. Extracted non-dimensional natural frequency for a simply supported cylindrical shell using present study matches very well with that of results reported by [10] as shown in Table 2. Non-dimensional natural frequency is given by  $\bar{\omega} = \omega \left( \frac{S^2}{h} \right) \sqrt{\frac{\rho^M}{E^M}}$ .

Table 2. Comparison of Non-dimensional natural frequency of the FG-CNTRC panel

| Modes | UD    |         |        | FG-X  |         |        |
|-------|-------|---------|--------|-------|---------|--------|
|       | [10]  | Present | %Diff. | [10]  | Present | %Diff. |
| 1     | 17.85 | 17.79   | 0.33   | 21.24 | 21.56   | 1.51   |
| 2     | 22.07 | 22.76   | 3.12   | 25.10 | 26.08   | 3.90   |
| 3     | 33.29 | 34.79   | 7.74   | 35.94 | 37.76   | 5.06   |
| 4     | 51.77 | 53.98   | 4.26   | 54.54 | 57.11   | 4.71   |
| 5     | 65.12 | 63.80   | 2.02   | 76.76 | 75.95   | 1.06   |

### 5. Results and discussion

The present study deals with the buckling and dynamic behavior of the FG-CNTRC cylindrical panel under thermal load. Effect of different geometrical parameters, CNTs volume fraction and boundary constraints on the buckling behavior of the panel is investigated. Further, the free vibration behavior of the panel exposed to a thermal environment is also addressed. Throughout the analysis, a cylindrical panel with thickness (h)=0.001m, thickness ratio (S/h)=100 and curvature ratio (R/S)=2 is considered otherwise it is mentioned. In the present study, poly-methyl methacrylate known to be PMMA, is taken for the matrix material with material properties  $E_m = (3.52 - 0.0034T)$  GPa,  $\nu_m = 0.34$  and  $\alpha_m = 45(1 + 0.0005\Delta T)10^{-6}/K$ . In calculation of elasticity modulus of matrix  $T = T_0 + \Delta T$  where  $T_0 = 300$  K is the reference temperature. For reinforcement, (10, 10) armchair single-walled carbon-nanotubes (SWCNT) is selected. Poissons ratio, shear modulus, Elasticity modulus and thermal expansion coefficient of SWCNT are assumed to be temperature dependent. To investigate the influence of temperature independent (TID) and temperature dependent (TD) properties on the buckling and dynamic behavior of the panel, an investigation is carried out on the thermo-mechanical properties of (10, 10) armchair SWCNT as a function of temperature using a third order interpolation [19]. Change in thermo-mechanical properties of (10, 10) armchair SWCNT with temperature for a range of  $300K \leq T \leq 700K$  are as follows

$$\begin{aligned}
 E_{11}^{CNT}(T) &= 6.3998 - 4.338417 \times 10^{-3}T + 7.43 \times 10^{-6}T^2 - 4.458333 \times 10^{-9}T^3 [TPa] \\
 E_{22}^{CNT}(T) &= 8.02155 - 5.420375 \times 10^{-3}T + 9.275 \times 10^{-6}T^2 - 5.5625 \times 10^{-9}T^3 [TPa] \\
 G_{12}^{CNT}(T) &= 1.40755 + 3.476208 \times 10^{-3}T - 6.965 \times 10^{-6}T^2 - 4.479167 \times 10^{-9}T^3 [TPa] \\
 \alpha_{11}^{CNT}(T) &= (-1.12515 + 0.02291688T - 2.887 \times 10^{-5}T^2 + 1.13625 \times 10^{-8}T^3)\alpha_0 \\
 \alpha_{22}^{CNT}(T) &= (5.43715 + 9.84625 \times 10^{-4}T - 2.9 \times 10^{-7}T^2 + 1.25 \times 10^{-11}T^3)\alpha_0 \\
 \nu_{12}^{CNT}(T) &= 0.175 \\
 \alpha_0 &= 10^{-6}/K
 \end{aligned}
 \tag{12}$$

Eight different pattern of CNTs distribution (FG-X, FG-O, FG-V, FG-Λ, FG-V-O-Λ, FG-Λ-O-V, FG-V-X-Λ and FG-Λ-X-V) and volume fraction (0.12, 0.17, 0.28) are considered to analyze the influence of functional grading and volume fraction of CNTs on the buckling and free vibration characteristics of CNTRC cylindrical panel. The CNT efficiency parameters,  $\eta_i$  for different volume fractions are  $\eta_1 = 0.137$  and  $\eta_2 = 1.022$  for  $V_{CNT}^* = 0.12$ ,  $\eta_1 = 0.142$  and  $\eta_2 = 1.626$  for  $V_{CNT}^* = 0.17$  and  $\eta_1 = 0.141$  and  $\eta_2 = 1.585$  for  $V_{CNT}^* = 0.28$  and  $\eta_2 = \eta_3$ . Panel is analyzed with four different boundary constraints CCCC, SSCC, SSSS and CCFC (where C- clamped, S- simply supported and F- free). The first letter in these boundary constraints is associated with forefront curved edge at x=0 in order ( Table 3).

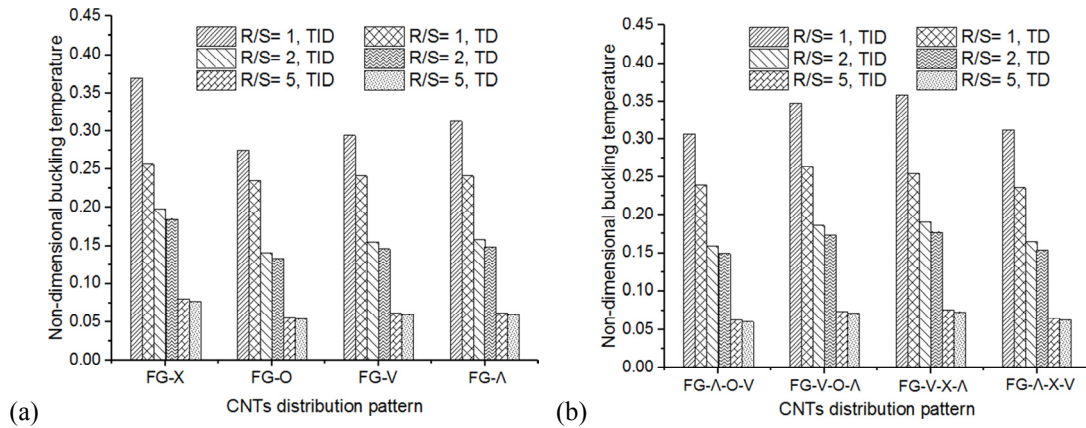


Fig. 5. Effect of curvature ratio and CNTs grading patterns on buckling strength of the panel (a) FG-X, FG-O, FG-V and FG-Λ (b) FG-Λ-X-V, FG-V-O-Λ, FG-Λ-O-V and FG-V-X-Λ

Panel with all restrained edges is simulated using CCCC boundary constraints. Further, combined effect of simply

Table 3. Different structural boundary constraints investigated

| Structural and boundary constraints            |   |                               |  |
|--|---|-------------------------------|--|
| CCCC   | SSCC  | SSSS                          | CCFC   |
| $x=0, L; u=v=w=0$<br>$\theta_x = \theta_y = 0$ | $x=0; u=w=0; \theta_y = 0$<br>$x=L; u=v=w=0$<br>$\theta_x = \theta_y = 0$ | $x=0, L; u=w=0; \theta_y = 0$ | $x=0; u=v=w=0$<br>$\theta_x = \theta_y = 0$    |
| $y=0, S; u=v=w=0$<br>$\theta_x = \theta_y = 0$ | $y=S; v=w=0; \theta_x = 0$<br>$y=0; u=v=w=0$<br>$\theta_x = \theta_y = 0$ | $y=0, S; v=w=0; \theta_x = 0$ | $y=0, S; u=v=w=0$<br>$\theta_x = \theta_y = 0$ |

supported and clamped edges is analyzed using SSCC boundary constraints. Finally, the influence of free edge on the buckling behavior of the panel is investigated using CCFC boundary constraints. Non-dimensional critical buckling temperature used in the present analysis is given below:

$$T_{c_r}^* = T_{cr} \times a_0 \times 10^{-3} \tag{13}$$

To study the influence of different CNTs grading pattern and curvature ratios on the buckling behavior, a panel is analyzed under eight different CNTs grading pattern along with three curvature ratios. Fig. 5 shows the non-dimensional buckling temperature for different cases analyzed. It is observed from Fig. 5 that CNTs distribution pattern significantly influences the buckling strength of the panel under thermal load. FG-X pattern is observed to have highest buckling strength of the panel. Fig. 5 also depicts that the hybrid CNTs distribution such as FG-V-O-Λ, FG-Λ-O-V, FG-V-X-Λ and FG-Λ-X-V, provides better buckling strength compared to FG-O, FG-V, and FG-Λ. Further, the influence of curvature ratios on the buckling strength is found to be significant. Thus, the panel with curvature ratio(R/S) of 1 is found to have the highest buckling strength. This is due to the fact that moment of inertia of the panel changes with the curvature ratio. Effect of temperature dependent properties on the buckling strength is also noted in Fig. 5. Material properties of the panel deteriorate with temperature thus, reduces its stiffness. Influence of temperature dependent properties on the buckling strength is highly significant at lower curvature ratio. As the curvature ratio of the panel increases the effect of temperature dependent properties on the buckling strength of the panel is found to be minimal.

Fig. 6 depicts the influence of thickness ratio, temperature dependent properties and CNTs distribution pattern on the buckling strength of the panel. It is found from Fig. 6 that as the thickness ratio increases, the buckling strength

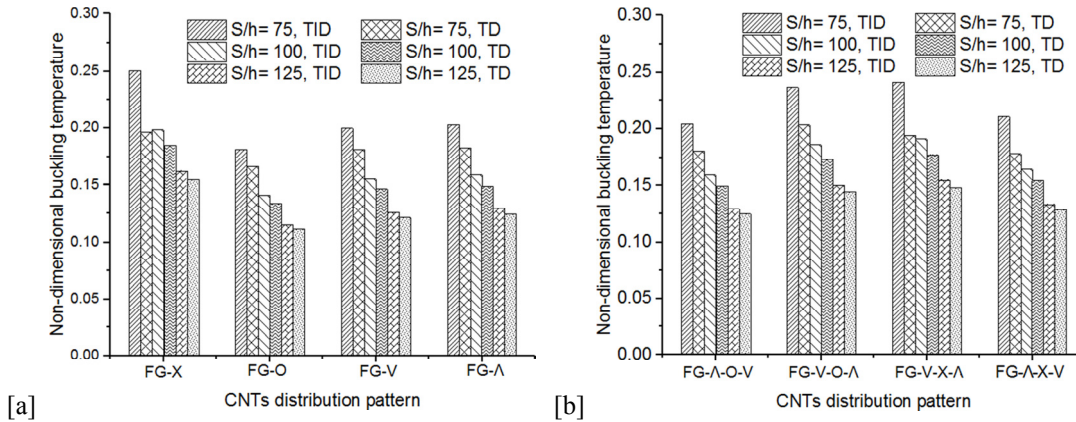


Fig. 6. Effect of thickness ratio and CNTs grading patterns on buckling strength of the panel (a) UD, FG-X, FG-O, FG-V and FG-Λ (b) FG-Λ-X-V, FG-V-O-Λ, FG-Λ-O-V and FG-V-X-Λ

of the panel decreases. This is due the fact that the panel loses its stiffness as the thickness ratio increases. Again it is seen from Fig. 6 that FG-V-O-Λ, FG-Λ-O-V, FG-V-X-Λ and FG-Λ-X-V provides better buckling strength compared to FG-O, FG-V and FG-Λ irrespective of thickness ratio. Thus, distribution of CNTs above and below the central plane plays a significant role in determining the buckling strength. Deterioration of the properties with temperature is also observed in Fig. 6 irrespective of the functional grading pattern. Influence of temperature dependent properties on the buckling strength is less at higher thickness ratio.

Structural boundary constraints affect the buckling strength of the panel under thermal load and same is depicted in Table 4. As expected, the panel with CCCC boundary constraints shows less buckling strength compared to other boundary constraints. This is due the fact that the panel with CCCC boundary constraints develop high thermal stresses which generates high membrane forces making panel to buckle at lower temperature. Panel with CCFC boundary constraints shows higher buckling strength compared to CCCC boundary constraints. This is mainly due to, free edge associated with the CCFC boundary constraints which allows some stress to relieve. Again the influence of temperature dependent properties on the buckling strength of the panel is observed under different boundary constraints. Effect of temperature dependent properties is more significant incase of SSSS boundary constraints. Table 4 also shows that the hybrid functional grading analyzed (FG-V-O-Λ, FG-Λ-O-V, FG-V-X-Λ and FG-Λ-X-V) provides more buckling strength irrespective of boundary constraints.

Table 4. Effect of boundary constraints and CNTs grading patterns on buckling strength of the panel

| CNTs distribution | CCCC  |       | SSCC  |       | SSSS  |       | CCFC  |       |
|-------------------|-------|-------|-------|-------|-------|-------|-------|-------|
|                   | TID   | TD    | TID   | TD    | TID   | TD    | TID   | TD    |
| FG-X              | 0.198 | 0.185 | 0.278 | 0.236 | 0.493 | 0.307 | 0.233 | 0.200 |
| FG-O              | 0.141 | 0.133 | 0.199 | 0.184 | 0.391 | 0.285 | 0.159 | 0.146 |
| FG-V              | 0.155 | 0.146 | 0.220 | 0.204 | 0.443 | 0.293 | 0.177 | 0.161 |
| FG-Λ              | 0.159 | 0.149 | 0.229 | 0.209 | 0.417 | 0.292 | 0.180 | 0.163 |
| FG-Λ-O-V          | 0.159 | 0.150 | 0.228 | 0.209 | 0.435 | 0.288 | 0.182 | 0.165 |
| FG-V-O-Λ          | 0.186 | 0.174 | 0.260 | 0.242 | 0.511 | 0.313 | 0.217 | 0.194 |
| FG-Λ-X-V          | 0.191 | 0.178 | 0.269 | 0.233 | 0.486 | 0.305 | 0.222 | 0.197 |
| FG-V-X-Λ          | 0.165 | 0.155 | 0.233 | 0.210 | 0.429 | 0.285 | 0.190 | 0.171 |

Influence of volume fraction of CNTs and functional grading pattern on the buckling strength of the panel is seen in Table 5. It is observed that, as the CNTs volume fraction increases, the stiffness of the panel increases thus offers higher buckling strength. Panel with CNTs volume fraction of 0.28 shows higher buckling strength irrespective of the functional grading pattern. Even under different volume fraction, panel with FG-V-O-Λ, FG-Λ-O-V, FG-V-X-Λ and



FG-A-X-V, shows better buckling strength compared to FG-O, FG-V and FG-A. Influence of temperature dependent properties on the buckling strength of the panel is more significant at high CNTs volume fraction.

Table 5. Effect of volume fraction of CNTs on buckling strength of the panel

| CNTs distribution | 0.12  |       | 0.17  |       | 0.28  |       |
|-------------------|-------|-------|-------|-------|-------|-------|
|                   | TID   | TD    | TID   | TD    | TID   | TD    |
| FG-X              | 0.198 | 0.185 | 0.208 | 0.195 | 0.257 | 0.204 |
| FG-O              | 0.141 | 0.133 | 0.147 | 0.140 | 0.176 | 0.165 |
| FG-V              | 0.155 | 0.146 | 0.162 | 0.154 | 0.198 | 0.183 |
| FG-A              | 0.159 | 0.149 | 0.166 | 0.157 | 0.201 | 0.186 |
| FG-A-O-V          | 0.159 | 0.150 | 0.167 | 0.158 | 0.202 | 0.187 |
| FG-V-O-A          | 0.186 | 0.174 | 0.194 | 0.183 | 0.239 | 0.217 |
| FG-A-X-V          | 0.191 | 0.178 | 0.200 | 0.189 | 0.248 | 0.209 |
| FG-V-X-A          | 0.165 | 0.155 | 0.172 | 0.162 | 0.207 | 0.190 |

Table 6 depicts the influence of thermal load (Buckling temperature function), CNTs distribution pattern and temperature dependent properties on the natural frequencies of the panel. It can be seen that the natural frequency of the panel decreases drastically at 95% of buckling temperature irrespective of the CNTs functional grading. This is due to the fact that panel losses its stiffness at a higher temperature. Effect of temperature dependent properties on the free vibration frequencies is also observed from Table 6. Wherein it is observed that the deterioration of material properties with temperature reduces the frequencies of the panel. Panel with FG-X CNTs distribution noted to have a frequency of 2883Hz which reduces to 1610Hz at 95% of buckling temperature under temperature independent properties and same frequency under temperature dependent properties further reduces to 1337Hz. Thus it is found that panel under thermal load with temperature dependent properties influences the dynamic behavior significantly. Hybrid CNTs functional grading gives higher frequency compared to FG-O, FG-V, and FG-A.

Table 6. Effect of Thermal load and temperature dependent properties on free vibration frequencies (Hz)

| CNTs distribution | Mode number | At ambient temperature | TID          |              | TD           |              |
|-------------------|-------------|------------------------|--------------|--------------|--------------|--------------|
|                   |             |                        | 55% $T_{cr}$ | 95% $T_{cr}$ | 55% $T_{cr}$ | 95% $T_{cr}$ |
| FG-X              | 1           | 2883                   | 2576         | 1616         | 2642         | 1308         |
|                   | 2           | 3059                   | 2615         | 1667         | 2692         | 1316         |
|                   | 3           | 3367                   | 2707         | 1969         | 2799         | 1849         |
| FG-O              | 1           | 1773                   | 1615         | 1315         | 1641         | 1283         |
|                   | 2           | 2033                   | 1730         | 1339         | 1769         | 1285         |
|                   | 3           | 2446                   | 1938         | 1466         | 1996         | 1286         |
| FG-V              | 1           | 2043                   | 1856         | 1406         | 1889         | 1293         |
|                   | 2           | 2269                   | 1940         | 1437         | 1985         | 1297         |
|                   | 3           | 2640                   | 2100         | 1622         | 2163         | 1371         |
| FG-A              | 1           | 2047                   | 1857         | 1450         | 1891         | 1375         |
|                   | 2           | 2290                   | 1954         | 1494         | 2001         | 1390         |
|                   | 3           | 2693                   | 2141         | 1597         | 2209         | 1414         |
| FG-V-O-A          | 1           | 2125                   | 1930         | 1465         | 1965         | 1346         |
|                   | 2           | 2350                   | 2013         | 1493         | 2060         | 1361         |
|                   | 3           | 2724                   | 2175         | 1655         | 2240         | 1424         |
| FG-A-O-V          | 1           | 2615                   | 2353         | 1578         | 2404         | 1249         |
|                   | 2           | 2804                   | 2403         | 1593         | 2465         | 1278         |
|                   | 3           | 3132                   | 2512         | 1907         | 2590         | 1622         |
| FG-V-X-A          | 1           | 2737                   | 2457         | 1612         | 2514         | 1308         |
|                   | 2           | 2920                   | 2503         | 1658         | 2572         | 1328         |
|                   | 3           | 3239                   | 2608         | 1895         | 2692         | 1694         |
| FG-A-X-V          | 1           | 2212                   | 2004         | 1464         | 2042         | 1288         |
|                   | 2           | 2430                   | 2078         | 1488         | 2129         | 1321         |
|                   | 3           | 2793                   | 2225         | 1680         | 2293         | 1435         |

## 6. Conclusion

Present paper deals with the buckling and free vibration behavior of FG-CNTRC cylindrical panels exposed to uniform temperature field using finite element method. The cylindrical panel analyzed is assumed to have temperature dependent properties. A detailed study is carried out to analyze the influence of temperature dependent properties, geometrical parameters, CNTs grading pattern, volume fraction and boundary constraints on the combined buckling and free vibration behavior of the panel. From the analysis, it is found that thermal buckling strength of the panel is significantly influenced by the temperature dependent properties and CNTs grading patterns. FG-X type of CNTs grading pattern gives better buckling strength than the other types as it has more CNTs grading away from the neutral plane. Hybrid CNTs functional grading gives higher buckling strength and frequencies compared to CNTs grading of type FG-O, FG-V, and FG- $\Lambda$ . It is also found that the natural frequencies of the FG-CNTRC panel decrease with increase in temperature due to thermal stress and temperature dependent properties and found to be significant at a temperature close to buckling temperature.

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